Department of Economics and Finance

Chair in Advanced Corporate Finance

Performance Measurement in Mutual Fund Analysis

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# TABLE OF CONTENTS

<table>
<thead>
<tr>
<th>Chapter</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>TABLE OF CONTENTS</td>
<td>ii</td>
</tr>
<tr>
<td>INTRODUCTION</td>
<td>3</td>
</tr>
<tr>
<td>CHAPTER I: Background and Literature Review</td>
<td>4</td>
</tr>
<tr>
<td>Performance Measurement</td>
<td>4</td>
</tr>
<tr>
<td>Stock Picking and Market timing</td>
<td>5</td>
</tr>
<tr>
<td>Time Varying Risk Factor Models</td>
<td>5</td>
</tr>
<tr>
<td>CHAPTER II: METHODOLOGY</td>
<td>6</td>
</tr>
<tr>
<td>Capita Asset Pricing Models (CAPM) &amp; Quadratic CAPM</td>
<td>6</td>
</tr>
<tr>
<td>Multifactor Models</td>
<td>7</td>
</tr>
<tr>
<td>Time varying approach</td>
<td>7</td>
</tr>
<tr>
<td>Autoregressive Models</td>
<td>8</td>
</tr>
<tr>
<td>CHAPTER III: RESULTS</td>
<td>9</td>
</tr>
<tr>
<td>Rolling analysis</td>
<td>9</td>
</tr>
<tr>
<td>Time Varying Analysis</td>
<td>10</td>
</tr>
<tr>
<td>CONCLUSION AND IMPLICATION</td>
<td>12</td>
</tr>
<tr>
<td>REFERENCES</td>
<td>14</td>
</tr>
</tbody>
</table>
INTRODUCTION

Why should we study mutual funds? They “simply” manage $30 trillion and growing.

![Percentage of U.S. mutual fund assets by type of fund](image)

Figure 1: Total Worldwide Mutual Fund assets. *Percentage of total net assets, year-end 2013*. Source: International Investment Funds Association

Mutual funds allow individuals to become investors by creating a system in which funds are pooled and then invested on their behalf in a litany of assets. There are numerous advantages available to investors who entertain the idea of mutual funds. One can buy a mutual fund’s share and gain the same access to a diversified portfolio. This action decreases the amount of risk associated with buying a variety of individual securities. Mutual fund analysts and managers overcome the large amounts of time and limited resources needed by an individual to optimize the asset allocation. They research information and execute their strategies. Using the *Center for Research in Security Prices (CRSP) Survivor Bias Free US Mutual Fund Database*, the following dissertation examines their performance. Mutual funds broke-out during the ‘80s when they experienced farfetched returns. Despite the introduction of *separate account*¹ and *exchange traded funds*² as valid alternatives, the industry is still growing, accounting for a market value for trillions of dollars, with more than ten thousand mutual funds. In spite of the constant growth of the industry, investors’ age distribution is not persistent.

Throughout the analysis of different risk factors we will identify the finest models to explain mutual fund performance. Moreover, we will test if mutual fund managers posses advantageous skills, which, in theory would allow them to outperform competitors. Therefore, we will try to answer whether a coefficients time series analysis can, and if so in which case, improve the precision of classical models. The dissertation aims to deliver a precise analysis as well as to clearly convey intuitions and concepts that may be foreign to audiences of differing backgrounds.

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¹ Investment account that a financial advisor uses to buy assets. The financial advisor pools money from different subjects. Differently from the mutual fund case, these subjects do not posses any security.

² A derivative traded on an exchange that differently from the mutual fund does not posses any net asset value (NAV).
CHAPTER I: Background and Literature Review

Since the early ‘70s, countless numbers of researches have been done in order to fully understand and explain the performance of mutual funds and their impact on modern economy.

Performance Measurement

Jensen (1968) studied mutual funds performance between 1945 and 1964 and was the first to use the alpha measure to evaluate mutual funds. He observed that previous authors were too concentrated on “relative measure of performance”, trying to rank performance to make better investment decisions. However, he spotted the lack of “absolute standard” to which the ranked performance should then be compared. Through his study of (only) 115 mutual funds, he observed that, on average, mutual funds are unable to outperform their index. He essentially implied that the average investor would have been better off with a buy and hold strategy.

Sharpe, W. (1992) studied the importance of asset allocation, as an efficient instrument to drastically improve investment decision-making, especially when a multitude of mutual funds is involved. He included in his work a class factor model, which he applied to study the performance of open-end mutual funds between 1985 and 1989, concluding that such a model can be employed in investment choices to reach financial objectives in a cost-effective way.

Grinblatt, M., Titman, S., Wermers, R. (1995) discussed the fluctuating mutual funds investment behavior and the different realized strategies. The authors observed from their results that the average mutual fund, correctly implementing a momentum strategy, outperform many other funds. During the same period Gruber, M. (1996) claimed that the NAV did not include a valuation of managers’ ability, thus being not able to predict performance and, consequently, funds flows. Through his work, the author claimed that mutual funds investors were more rational than previously thought. The author defined these investors as sophisticated clientele and counterpoised it to a disadvantaged one, hypothesis consistent with some empirical facts. To increase the precision of precedent studies, Ferson, W., Schadt, R. (1996) tried to incorporate public information into the performance evaluation process; an approach they named conditional performance evaluation. They presumed that if any fund manager only used present public information, he/she could not directly obtain abnormal returns. The authors confirmed CAPM and four factor model results, but they also argued that their conditional model is able to explain the alphas better. With their work, the authors suggested to study mutual funds performance implementing a conditional public information variable, to analyze investment performance in future research with.

Carhart, M. (1997), through the use of a survivorship bias free database, showed that common factors in the returns of stocks could describe most mutual funds returns. The author criticized previous approaches to the topic, arguing that past results were characterized by a momentum effect, not taken adequately into account. Moreover, he argued that individual funds could not outperform any benchmark using a momentum

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3 The reasons behind the disadvantage could be mere investor inexperience, institutional barriers, or tax regimes.
investment strategy. The author also confirmed that his results were consistent with the market efficient hypothesis.

**Stock Picking and Market timing**

Wermers, R., (2000) evidence proved a significant stock picking ability in high-turnover funds, which explains the higher return levels than those of low-turnover funds. Kacperczyk, M., Seru, A., (2007) claimed that the power of information is not in its possession but in the ability of the manager to use it. From this basic philosophy the authors crafted a model to evaluate the interaction between public information and managers’ skills. The author found proof that the managers relying less on public information were the ones that actually performed better.

Duan, Y., Hu, G., Mclean, D. (2009) showed that managers succeed at stock picking only with idiosyncratic stocks and that the number of managers good at stock-picking is decreasing over time. Baker, M., Litova, L., Wachtera, J., Wurglera, J. (2010) found that, on average, at the announcement of earning, what the fund has bought slightly outperform what the fund has sold. Funds trades could indeed predict announcements and earning per share surprises. They suggested that skilled managers could pick the right stocks throughout a proficient fundamental analysis and good understanding of the sector. Berk, J., Van Binsbergen, J. (2014), using a flow added value variable proved, differently from many other works, that managers do show specific investment skills. Furthermore, and surprisingly, the authors demonstrated that investors are, on average, able to identify those skills and actually reward them. A result that is consistent with Berk, J., Green (2004). Moreover, Berk, J., Van Binsbergen, J. (2014) argued that the alpha embedded investor rationality and market efficiency.

**Time Varying Risk Factor Models**

Mamaysky, H., Spiegel, M., Zhang, H., (2004) claimed that even though assets returns can be described by a model with time-invariant alphas and betas, a portfolio, especially one actively managed, cannot. The authors also analyze that, in order to do market timing, fund managers vary the risk exposure of their portfolios periodically. Mamaysky, H., Spiegel, M., Zhang, H., (2004) proposed a Kalman filter model to estimate the historical time series of funds alphas and betas. Compared to a classical rolling window OLS, their model performed better in tests, both in sample and out of sample but it results also more unreliable. Moreover, the estimates of the Kalman filter dynamic parameter offers a way to categorize funds on their strategy and also to understand their source of profits or losses. The main assumption of this paper was that portfolio holdings are driven by an unknown variable, which track an AR (1) process. This assumption made possible to use past changes in alphas and betas to perform good forecast; the authors said that if this assumption were proved to be true their model would better explain mutual fund returns than static OLS model.

Chiarella, C., Dieci, R., He, X., (2010) showed that changes in agents’ behavior led to changes in market portfolio, asset prices and returns, and time-varying betas. Moreover, they proved CAPM betas to be stochastic and thus implied a lack of explanatory power a time-varying CAPM based on rolling windows may have, is actually due to the underlying estimation technique, rather than the model assumptions.
CHAPTER II: METHODOLOGY

Capita Asset Pricing Models (CAPM) & Quadratic CAPM

Despite the assumption made by Sharp, W. (1964), we will move on from its theoretical background. We start from the Single index model regression:

\[ r_i - r_f = \alpha_i + \beta_{mkti}(r_m - r_f) + \varepsilon_i \]  

(1)

and moving to its timing series regression:

\[ r_{it} - r_{ft} = \alpha_i + \beta_{mkti}(r_{mt} - r_{ft}) + \varepsilon_{it} \]  

(2)

where \( \alpha_i \) is the outperform of what was predicted by the CAPM and \( \beta_{mkti} \) is the sensitivity measures of a fund to market movements assuming that \( \varepsilon_{it} \sim N(0, \sigma^2) \) in (1) and (2). Considering \( \beta_{mkti} \) in (2) as time dependent, and assuming that managers are willing to change their risk exposure depending on their personal expectations, we can write the \( \beta_{mkti} \) as a function in the form:

\[ \beta_{mkti}(t) = \beta_i + \gamma_i(r_{mt} - r_f) \]  

(3)

Both the \( \beta_{mkti}(t) \) and the \( \beta_i \) in (3) manage to explain this tendency to the risk. The \( \beta_i \), in particular, is a fixed parameter that refers to the investor’s risk propensity, when the expectations of the return match those of the market. It should also be clear that managers want to increase their exposure to the market risk when they expect a higher market return, and reduce it when the expected \( r_{mt} \) is lower. Indeed, \( r_{mt} \) is just the result of the investment in the index or in the market portfolio. In (3) the term \( \gamma_i \), if positive, indicates a manager’s superior timing ability. If we plug the equation (2) into the equation (3), we generate the following quadratic model:

\[ r_{it} - r_{ft} = \alpha_i + \beta_i(r_{mt} - r_f) + \gamma_i(r_{mt} - r_f)^2 + \varepsilon_{it} \]  

(4)

where, \( \alpha_i \) represents the outperform but interpreted as the manager’s securities selection ability (also known as stock picking), \( \beta_i(r_{mt} - r_f) \) is the extra return part due to market movements and \( \gamma_i(r_{mt} - r_f)^2 \) measures the influence of market timing on the total return. Notice that a manager owns a superior timing ability if the term \( \gamma_i \) is positive. Consequently, \( \alpha_i \) in equation (4) indicates how much better (worse) the manager did than CAPM predicted as also Cesari, R., Panetta, F. (2001) analysed with a similar model on Italians mutual funds. In other words the \( \alpha_i \) is the vertical distance of the return/beta combination from the security market line. Moreover, we understand from this model that to test market timing abilities we only look at the statistical significance of term \( \gamma_i \) indeed we run a much more simple analysis of the one of Henriksson, R., Merton, R., (1981). Is the term \( \alpha_i \), in equation (4), the manager’s selectivity alone?

No, it is not. The term \( \alpha_i \) measures outperform the market together with the manager’s selectivity. Hence, it is helpful to understand how to precisely measure the manager’s selectivity, so, in order to retrieve the raw value of the stock picking, it’s necessary to go deeper into the analysis.
Multifactor Models

Fama, E., French, K. (1993) identified three main factors for stock returns: a market factor, a size factor and a book-to-market ratio factor. Also, taking into consideration the analysis of Carhart, M. (1997 we will test the following model:

\[ r_{it} - r_f = \alpha_i + \beta_{mmt}(r_{mt} - r_f) + \beta_{SMB}(SMB) + \beta_{HML}(HML) + \beta_{MOM}(MOM) + \varepsilon_i \]

In Fama, E., French, K. (1993), the three factors appear to describe average returns on stocks properly. The choice of these specific variables was dictated by the experience, the anomaly of the market, and not by the theory, still not completely able to specify the common factors in returns. However, if those factors achieve to seize the cross-action of average return they can be used to understand the residual anomalies on the market and to improve portfolio selection. Now, going back to the quadratic analysis, we need a multi-factor model that uses more than one portfolio to capture systematic risk.

Regardless, the specific factors introduced by the literature, apart from the market portfolio, each additional portfolio can be thought as the risk factor itself, which is part of the systematic risk, previously not captured. Let us look at the following equations:

\[ r_{it} - r_f = \alpha_i + \beta_i(r_{mt} - r_f) + \gamma_i(r_{mt} - r_f)^2 + \beta_{MOMt}(MOM) + \varepsilon_i \]

\[ r_{it} - r_f = \alpha_i + \beta_i(r_{mt} - r_f) + \beta_{MOMt}(MOM) + \varepsilon_i \]

The models above are periodically re-balanced into those stocks exhibiting the best momentum that will produce higher expected returns. However, it is also necessary to test for multicollinearity, and autocorrelation to reach a consistent conclusion. From these regressions we can analyse the manager’s selectivity, by considering their \( \alpha_i \). In this prospective, the aim of the research is to point out whether the presence of stock picking is statistically significant and, in primis, if it has a true influence in explaining the portfolio return. Moreover, we can isolate the impact of the market-timing component of performance, \( \gamma_i(r_{mt} - r_f)^2 \), which could be strongly influenced by the momentum factor. From this other prospective, we should start from the alpha analysis, in case of statistically significant results.

Time varying approach

If any fund dynamically regulates its investment selection as a reaction to variations in the financial environment, then, the constant coefficient models will largely be biased and might even be misdirected. In security analysis, as much as in company valuation, the precision of beta is crucial in both investment strategies decision and equities pricing. An essential feature of the beta-metric is the explanatory power it lends to assess portfolio risk and returns. Jan, K. (2011) used time-varying forecasts through an autoregressive process to form a predictive non-constant beta model. His results showed that the time varying beta captures persistence in financial data better than the constant beta framework developed by Sharpe, W. (1964). Moreover, Jan, K. (2011) showed that the time varying beta bounds the noise effect in high-frequency data. The author suggested the use of high frequency data, but, due to the nature of our research, we will use the same framework extending both the time lag and the time series, in order to maintain the background structure. When we examine time series in the Sharpe, W. (1964) framework, a key hypothesis is time invariance of the risk factors. However, since the financial market environment experiences continuous

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4A portfolio constructed by buying stocks that have recently done extremely well and selling those that have done extremely poorly. It is also called the prior 1-year (PR1YR) momentum portfolio.
change, it might not be realistic to accept that the parameters are constant. A method to evaluate this constancy is to calculate parameters over a fixed rolling window over the sample: in case parameters are constant, the rolling window should not be too far from the constancy value; while in case of non constancy, estimation should seize the volatility. In order to move from the risk factors analysis to the time series analysis of the most significant risk factors, we constructed a new database containing the betas time series of each of the precedent models through a beta rolling procedure. The window size of the OLS rolling regression, as suggested by the literature, is five year for each model and the betas are estimated on a monthly rolling base. Again, our database goes from January 2000 to December 2013, therefore, it contains a maximum of 168 observations for each fund; it could be less than 168, due to its intrinsic characteristic of being a survivorship bias free database, but we will constrain the time series in our new database to a minimum of 84 monthly observations.

**Autoregressive Models**

Before getting into the specifics of our time series analysis a small introduction on the autoregressive process would be essential. An autoregressive process of order $p$, AR ($p$) is defined as $\beta_t = m + \phi_1\beta_{t-1} + \phi_2\beta_{t-2} + \cdots + \phi_p\beta_{t-p} + \epsilon_t$ where $\epsilon_t \sim WN(0, \sigma^2)$. We can also safely assume $m = 0$, implying that the mean of the process is zero as well. An alternative way to represent AR ($p$) is to write it through a lag polynomial. Defying $L$ as the lag operator, we have $(1 - \phi_1L - \phi_2L^2 - \cdots - \phi_pL^p)\beta_t = \epsilon_t$. Therefore $\varphi(L)\beta_t = \epsilon_t$. From this polynomial, we can move to the Wold representation of the process. The Wold theorem tells us that every stationary process can be expressed as a linear combination, an infinite moving average, of White Noise (WN) processes $\beta_t = \frac{1}{\varphi(L)}\epsilon_t$ where the inverse of the $p$-order lag polynomial is an infinite polynomial. From the Wold representation, we can obtain moving average processes (MA), but estimating an infinite MA is impossible. Any truncation to a finite MA ($q$) would not be as accurate, though. However, it is possible to represent an infinite MA combining the AR ($p$) and the MA ($q$) into an ARMA ($p$, $q$). Basically, it is a generalization of a classic AR process with innovations following an MA process, or equivalently as a moving average auto-regressed on its past values:

$$\beta_t = \phi_1\beta_{t-1} + \phi_2\beta_{t-2} + \cdots + \phi_p\beta_{t-p} + \epsilon_t + \theta_1\epsilon_{t-1} + \cdots + \theta_q\epsilon_{t-q}$$

$$\varphi(L)\beta_t = \theta(L)\epsilon_t$$

Stationary conditions for an ARMA process are the same as those of an AR ($p$) one. In our research, we are now dealing with betas’ time series, therefore due to the beta nature of our analysis we will only study the AR (1) in the form; $\beta_t = +\phi_1\beta_{t-1} + \epsilon_t$ where $\epsilon_t \sim WN(0, \sigma^2)$. The order of the autoregressive parameter represents how far, in term of unit period, the information still affects the present value. It would not make much financial sense to consider an AR ($p$) with $p$ higher than one. The financial markets, usually, are able to absorb information quickly due to the competitive environment created by the characters in it. The unit period we are considering here is one month, therefore, considering an autoregressive process of order two for example, would indicate that the present information are influenced not only by the previous month’s values but also by the values of two periods before as if markets’ shocks and public information would not have been already metabolized by the previous month’s values.

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5 The roots of the above polynomial must be outside the unit circle
CHAPTER III: RESULTS

Rolling analysis

Given the information set analyzed, we will describe the procedure followed in order to implement the various models used. We built and test four different models: the traditional Capital Asset Pricing Model as well as the traditional Fama-French-Carhart Model and two modified version of the CAPM. The first one introduced a new parameter depending on the square of the market premium as of equation (4); the second one merged the classical version of the CAPM with the Carhart Factor, the Momentum one, as of equation (6). We decided to estimate time varying betas instead of the classical constant beta. This choice was led by both the willingness to perform a more reliable regression and the decision to build a database for the betas autoregressive analysis model, which will be explained at a later stage. In order to proceed with this kind of estimation, we built a rolling window of 60 months, 5 years as suggested by the literature, used to perform a multi linear regression of the models’ factors over the premium of each fund. Instead of getting only one beta per fund, we got 109 time varying betas for each. The timeframe of the analysis was originally made of 168 months observations, 13 years, which became 109 after the regression, as a consequence of the use of a 60 months window size. The same procedure was used for the alphas estimation. We then proceeded with a detailed comparison and analysis of the outputs. The p-value of each coefficient and the reliability, $R^2$ and Adjusted $R^2$, of each single regression were highly taken into consideration: the aim was to understand the forecasting power of each model for the U.S. mutual funds universe, 5796 funds, over the period (2000-2013) analyzed.

In order to present the results of the rolling beta we will not use a classical regression representation, as we would have more than 20,000 outputs considering all the results for all the 4 models. Reason why, we calculated the percentage of $P$-Value significance, for each parameter, in each model, as the ratio of the number of parameters statistically significant at a 5% level of confidence divided by the total number of parameters estimated in that specific model.

| Table 1: Parameters’ P-values percentage significance |
|---|---|---|---|---|---|
|   | $\alpha$ | $\beta$ Mkt | $\beta$ Mkt$^2$ | $\beta$ SMB | $\beta$ HML | $\beta$ MOM |
| CAPM | 12.68% | 83.65% | | | | |
| CAPM$^2$ | 12.00% | 83.05% | 9.72% | | | |
| CAPM Mom | 13.19% | 83.62% | | | 34.51% |
| Fama-French-Mom | 12.63% | 83.40% | 42.19% | 48.00% | 34.50% |

The first information that should grab the attention of the reader is the low percentage of significance of the alpha. In all the models the number of the significant alpha is never more than 13.19%: the result does support the absence of persistency in market outperforming for the industry. We can also see that there is no proof of market timing: even if most of the gamma coefficients of equation (4) are positive, less then 10% of them turns out to be statistically significant. Moreover, and maybe surprisingly, the Fama-French factor together with the momentum are not statistically significant in more than half of the parameters estimated suggesting that the market premium might absorbed their explanatory power.
**Time Varying Analysis**

We estimated an AR (1) on the beta time series obtained with the Rolling CAPM. The choice of keeping only one lag is justified by the monthly frequency of our betas: relevant shocks can be absorbed and processed by the market in that period and, therefore, fund managers can reassess the exposure to the market risk and rebalance their portfolios. In our opinion, it is indeed reasonable to assume that betas, at each time, can be inferred only from the previous period beta, this idea is also supported by the literature discussed. From our regression coefficients, the first result we can infer is about the stationary of the beta time series. We care about stationary because stationary betas, betas oscillating around their mean, suggest that the market exposure of the fund remains persistent, thus, we can assume that there is an underlying investment strategy to maintain its persistence. We have described the AR(1) results regardless of its relationship with the other modes analyzed in this work. However, it would make sense to relate this model with the others and observe its relative strength, especially when it comes to forecasting power. The question we ask is: how much can an AR(1) analysis on betas improve decisions made throughout investment strategies if actually used? And more notably, is it truly worth using?

To answer these questions we performed an analysis inspired by Reeves, J., Haifeng, W., (2010). In the paper, the authors compared three models of beta estimations: the Fama-MacBeth constant beta model, a simple constant beta model, and an autoregressive time-varying beta model. In order to do so, the authors estimated the betas from these three models on a database of daily stock returns and looked at the mean square error (MSE) of the obtained beta forecasts and the realized betas. Starting from their framework, what we have attempted to do is to use an analogous approach and adapted it to our needs; in particular, we had to use monthly fund returns and compare return forecasts, not beta. We estimated a constant beta model and a CAPM based on a rolling window technique. Then, we estimated an AR(1) on the betas obtained with the rolling window method. After the estimation phase, we computed beta forecasts \( \hat{\beta}_{mkt,t} \) for each model and used them to find return forecasts:

\[
\hat{r}_{t,t} = r_{f,t} + \hat{\beta}_{mkt,t} (r_{m,t} - r_{f,t})
\]

We compared these forecasts with the actual returns, and found the MSE, for each fund and each model as:

\[
MSE_i = \frac{1}{T} \sum_{t=1}^{T} (r_{i,t} - \hat{r}_{i,t})^2
\]

Where:
- \( T \) is the total number of forecasting period, twelve months in our case
- \( r_{i,t} \) is the observed return at time \( t \) for the \( i \)-th fund,
- \( \hat{r}_{i,t} \) is the return forecast at time \( t \) for the \( i \)-th fund.

We compared the obtained MSE over all funds and models and observed which specification had the smallest one. In particular, we compared the AR (1) MSE with the CAMP MSE, the AR (1) MSE with the Rolling MSE, and the Rolling MSE with the CAPM MSE. As far as the first comparison is concerned, on the whole sample, the AR (1) model can lead to an improvement on return forecasts for 28.1% of the funds in consideration; therefore, if those funds are taken into consideration during an

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6 The authors also looked at the mean absolute error (MAE) but since the results express the same concept we will only discuss the MSE comparison.

7 For the relevant equations of the used models the reader may look at the previous sections.
investment decision, an investor should be supported by an autoregressive analysis instead of just the CAPM. The comparison between the AR and the beta rolling sees the former prevailing in almost half of the sample (42.9%). Finally, looking at the CAPM MSE and the Rolling MSE, the latter cannot do better than the former in more than 19.7% funds. These results, with the CAPM model offering the best predictions, in most cases, are also perfectly aligned with those of Reeves, J., Haifeng, W., (2010) obtained for the beta forecasts. Furthermore, if we extend our analysis a little more, we can consider the sub-sample of stationary funds; in this instance there is a significant improvement in the autoregressive results, with an increase in its assessment power of almost 10%.

As far as the non-stationary funds are concerned, they may require another type of model to better explain them, an ARIMA class model is a case in point. We find it to be useful to give the reader a little preview on the ARIMA class of models. Dealing with Autoregressive Integrated Moving Average (ARIMA) models means dealing with a non-stationary process. An ARIMA model can be “simply” thought as an ARMA model where the stationary condition has not been met. Therefore, from an ARMA \((p, q)\), we move to an ARIMA \((p, d, q)\), where:
- \(p\) and \(q\) are the order of the AR component and the MA component respectively;
- \(d\) is the order of the unit root, which, basically, identifies the order of the non-stationary.

The starting point for an ARIMA identification is performing a unit roots test. Unit roots tests are essential in laying the foundation to interpret trends in financial time series. The trend, indeed, is the source of non-stationary and can be either deterministic, thus giving rise to trend-stationary processes (TS), or stochastic, in which case it can be difficult to disentangle from the series, as is the case of difference-stationary processes (DS). Seasonality is another important aspect to consider. Indeed, there may be patterns inside the trend of a non-stationary process, meaning that the trend may be changing over time periodically. In financial time series, seasonality may be related to changes in investors’ risk propensity, and thus be captured on an annual, monthly or weekly basis. Seasonality can also be linked to the economic cycle and cover a larger horizon. In both cases, it is evident how much an analysis of the seasonality can be useful and insightful for the type of research we are conducting, offering more powerful and predictive results. Estimating an ARIMA \((p, d, q)\) means we must identify its trend and cycle components, using a complex and demanding framework, which goes beyond the aim of this research. Several processes, in fact, can represent a time series, and choosing one over another is often left to the discretion of the researcher, rather than an objective criteria. Once an ARIMA is estimated, though, the steps of the analysis on the forecasts MSE are analogous to the ones already performed. Adding this last model can supply a fund manager or an investor a complete framework that assesses funds returns by classifying them according to each fund’s intrinsic characteristics and choosing the best prediction model for each of them. Therefore, our model only complements and supports the classic CAPM analysis.

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8 The roots of the characteristic polynomial were inside the unite circle
9 In this concern, Nelson, C., Plosser, C., (1982) tested whether many macroeconomic time series were DS or TS, determining a prevalence of the former.
CONCLUSION AND IMPLICATION

Our work has moved from the reproduction of classical risk factor models to the application of new methods for mutual fund selection.

The reproduction of previous works showed how models have changed over time and how the financial markets have absorbed their value. We observed that the CAPM is the best predictor for mutual fund return. The other models analyzed turned out to be inferior when compared on the explanatory power and also showed, on average, insignificance of their parameters. Due to the amount of estimations that has been run we feel required to underlying how the alpha, in all the models implemented, resulted on average statistically insignificant.

Moreover, we asked ourselves why all the parameters are on average not significant even if their factor loading are constructed without taking the systematic risk into consideration.

The first approach to the answer has been very straightforward: we repeated the analysis again and again until we were sure about the reliability of the results. Only then we thought that, maybe, the market efficiency could partially find an answer to the issue. Markets might not be fully efficient, as it has already been proved, but they are competitive for sure; this competitiveness results in the attempt to exploit all the best investment opportunities, after all there are no free lunch. Therefore, if any model is actually able to capture an investment opportunity, as the Fama-French has been proved to be, the agents in the market will start to implement that specific model in their investment decision process in a procedure that results in the tendency to efficiency of the financial markets itself.

We did see how the autoregressive analysis of the beta time series can improve the forecasting power of the CAPM for more than one third of the stationary process. What could be a concrete implication of our work? Could it simplify the investment decision process?

The following plot displays what an investor could use after the market risk exposure has been chosen.

Figure 2: Stationary desirable mutual funds 3D-Plot
Here we see a subset of the constellation of all the mutual funds of our sample. We focused on the stationary one with positive historical past performance, through which, once the market risk exposure has been decided, the investor could examine the different choices with the CAPM approach.

The question now is: why an investor should focus on this subsample?

After the financial disasters occurred in the last years, all the agents are more concerned about risk in general but also much more aware about the impact of market risk. If each agent and/or investor could precisely formulate his own market risk aversion\(^{10}\), this analysis could give him/her the chance to pick a fund that, since has been proved to be stationary over its beta time series, is expected not to move too far from its average values; condition that the investor looked for in the first place. Since the previous analysis is based on the stationary of the autoregressive beta process, the phi in the 3D plot is constrained to be less than one in module; an ARIMA identification is essential to push our research forward for the non-stationary betas. Due to the technicalities of the subject and the computational complexity of the topic, the betas estimation for the non-stationary mutual fund time series is left as an open question, in hopes of one day being pursued. If such analysis would create a framework similar to this one we could provide investor with a tool that, using the same underling model of choices, better assess the match between investor and market risk.

\(^{10}\) Notice that in this way we are not making any assumption about their risk propensity
REFERENCES


