Department of Economics and Management

Chair of Money and Banking

Unconventional monetary policies: a survey of quantitative and credit easing and methods of evaluation of the portfolio-rebalancing channel

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Introduction
The recent financial crisis has pushed the world into a deep recession. To give relief to the economy, many central banks lowered the short-term interest rates to zero and, in same time, they engaged unconventional monetary policies, such as forward guidance, quantitative easing and credit easing. Main target for policy makers was to give relief to the inter-bank and credit markets first, and then to the economy, preventing a further fall of lending activity, thus dampening the fall of inflation and output. Therefore, the role of the central banking has been increasing in importance during last years, especially because their policies can stabilize the financial and economic system during periods of turmoil. In fact, it is important to understand how policy makers conduct those policies and through what mechanisms they are able to affect the real economy. Moreover, central banks are investing more and more resources in developing models useful to analyze their policy and to forecast economic trends. In this context, central banks are developing more and more DSGE (dynamic stochastic general equilibrium) models for monetary policy evaluation, since they can help to identify sources of fluctuations, forecast monetary policy effects and make counterfactual experiments.

This work attempts to analyze the main balance sheet policies such as quantitative easing and credit easing, focusing on both theoretical features and empirical aspects, recalling past Japan, US and EU experiences. Furthermore, this work explores the implementation of DSGE models for the analysis of the different transmission channels of balance sheet policies. The first chapter describes conventional and unconventional monetary policies. Regarding the first ones, there will be an analysis about the main conventional instruments and the correlated transmission mechanism. After, the chapter analyses the zero-lower bound and the liquidity trap, situations that lead the central bank to pursue the unconventional monetary policies, such as forward guidance, quantitative easing and credit easing. The analysis goes deeper on the latter two, the so-called balance sheet policies, and on the correlated transmission mechanism. After the analysis is developed both under theoretical and empirical way, making a parallelism between direct/indirect quantitative and credit easing in theory and in practice. The chapter ends with a survey on main risks that the central bank faces when undertaking these policies. The second chapter explores the methods of evaluation of the unconventional monetary policies, focusing especially on DSGE models. After exploring briefly the historical evolution of modeling, the chapter analyses the main characteristics and the basic structure of standard DSGE models. Therefore, the analysis
continues on a particular elaborated DSGE model, which attempts to isolate the portfolio-rebalancing channel of unconventional transmission mechanism. The model is firstly analyzed under the mathematical aspect, showing the different equations that dominate the general equilibrium. Then, the study continues on the functioning behind the model and on the particular aspects that make this model different to other types of DSGE, such as the imperfect asset substitutability, the term premium structure and the feedback channel. Finally, the last part shows the main results, which shows the relevance of the portfolio-rebalancing channel as an instrument to boost real economy. Thus, this part exerts a comparison about the portfolio-rebalancing channel effects across the different balance sheet policies and explores other DSGE studies made on other transmission channels, such as bank-lending channel and liquidity premia channel.

To conclude, a brief survey on the limits of DSGE models is conducted. This work shows us that nowadays the DSGEs are able to isolate and study different transmission mechanism of monetary policies. However, they still simplify too much the reality and in the same time, their results are difficult to communicate to policy makers and to the public. Therefore, economists have to face a double challenge across these models, one is to make them more complex in order to represent better the reality, and another is to make them easier to understand and to communicate.
1. Conventional and unconventional monetary policies
This chapter opens the analysis, in the paragraph (1.1) with the study of the conventional monetary policy instruments and conventional transmission mechanism. Conversely, the paragraphs (1.2-1.3) explore the unconventional monetary policies in general and, more specifically, the balance sheet policies and their transmission mechanism. Therefore, the paragraphs (1.4-1.6) analyze separately direct quantitative easing, direct credit easing and indirect quantitative/credit easing policies in theory, focusing on their pro and cons and on their impact on real economy. Thus, the paragraphs (1.7-1.9) analyze those three policies in the real world, recalling the US, Japanese and EU experienced. Finally, the paragraph (1.10) explains main risks to which the central bank is exposed when undertaking those policies.

1.1 Conventional monetary policies
During normal economic and financial periods, central banks pursue conventional monetary policy, which aims first to set a target overnight interest rate in the interbank money market by signaling the desired policy rate. To make that policy rate effective central bank recurs to different instruments, such as open market operations, discount window, deposit facilities and minimum reserve requirements for credit institutions.

The open market operations are repos (repurchasing agreements) undertaken in the collateralized lending market through which the central bank increases or lowers reserves to the banking system in exchange of collateral, usually high-rated bonds. Through these operations, the central bank is able to influence the uncollateralized lending rate between banks, which is the interest rate that monetary-financial institutions pay when they borrow overnight-uncollateralized loans of reserves balances from each other. The central bank is able to set the overnight interest rate to any particular level because it stands ready to buy and sell unlimited amount of bank reserves. Since the central bank wants to minimize its balance sheet’s risk, eligible collaterals, like short-term government bonds, characterize open market operations.

Discount window allows monetary-financial institutions to borrow money from the central bank for a short-term period through the discount of some quantity of its securities. The interest rate charged by the central bank in this type of operations is higher than the one used in the open market operations and it is called discount rate. This rate determines the ceiling for the overnight interest rate.
Regarding the deposit facilities, the central bank pays an interest rate on required reserves and on excess reserves deposited by the banks. This interest rate is lower to the one used in the open market operations and determines the floor for the overnight interest rate.

Through these instruments, the central bank achieves the target to maintain price stability over the medium term. Indeed, the central bank can lower policy rate during economic downturns in order to promote lending activity and can increase it during economic upturns. This is possible because there are different channels through which those key interest rates are able to affect the real economy. The figure (1.1) lists five conventional monetary policy transmission channels:

1. Credit channel
2. Wealth channel
3. Balance sheet channel
4. Interest rate channel
5. Exchange rate channel

**Figure 1.1: Conventional monetary policy transmission channels**

![Monetary policy transmission channels](https://www.oenb.at/en/Monetary-Policy/How-Monetary-Policy-Works.html)

The credit channel regards the supply of credit made by the banks. If the central bank lowers the key interest rates, money market interest rates will decrease. Thus, banks have to pay lower interest rates on households’ deposits and their balance sheet will
improve as well. Since banks can refinance themselves more easily, they are more stimulated to make loans and so to increase the credit supply. Moreover, lowering the interest rates leads to a reduction of the default risk of banks’ counterparts, which let the banks to grant more loans. More loans lead to more investment, consumption and a higher inflation.

The wealth channel regards the price of assets like stocks and real estate. If the central bank lowers the key interest rates, stock and real estate prices will increase. Stock prices increase since investors discount future dividends with a lower interest rate. Real estate prices increase because mortgage loans are cheaper, thus increasing the demand for housing. Higher stock and real estate prices increase households’ wealth, leading to a higher consumption and so to a higher aggregate demand and inflation. Higher stock prices make easier for firms to raise capital, since they earn more capital per share issued, leading to higher investment spending and so boosting aggregate demand and inflation as well.

The balance sheet channel regards the role of collateral of the assets. If the central bank lowers the key interest rates, net assets in the balance sheet increase. Thus, the value of collateral to take out loans increases as well, increasing lending and investment spending, hence aggregate demand and inflation.

The interest rate channel regards the short-term interest rates. If the central bank lowers the key interest rates, short-term interest rates will decrease as well. Therefore, real interest rates and the cost of capital falls, leading to lower saving and higher investment, boosting aggregate demand and so inflation.

The exchange rate channel regards the exchange rate of the central bank’s currency relatively to foreign currencies. If the central bank lowers the key interest rates, this will lead investors to sell domestic currency for foreign currencies. The outflow of funds from domestic country to other countries leads to a depreciation of the foreign currency. Thus, domestic goods become less expensive than imported goods, thus becoming more attractive. The demand for domestic goods increases, boosting aggregate output and inflation. The impact of this channel depends on the openness of the economy, more it is open and more this channel has effects on the economy.
1.2 Liquidity trap and unconventional monetary policies

In normal economic and financial conditions, the central bank can affect the real economy by modifying the key interest rates and recurring to the four instruments explained in paragraph (1.1) to make those interest rates effective. During economic downturn periods, the central bank can lower the key interest rates to zero to boost the real economy. If the central bank fails to achieve an increase of aggregate demand and inflation, the real economy falls in a situation called “Zero Lower Bound”. In the ZLB, the central bank cannot stimulate anymore the economy lowering interest rates, since they cannot go any lower. The ZLB problem causes the “Liquidity Trap”, concept originally conceived by Keynes (1936), who described it as a situation in which economic agents prefer holding cash instead of borrowing money at very low interest rate (for example due to negative future expectations), therefore pushing down investment, consumption, aggregate demand and inflation. Thus, any injection of money into the system would fail to achieve an increase of the macro-economic variables. Once in a liquidity trap, the economy enters in a deflationary spiral where real interest rates increase even further and where conventional monetary policy instruments lose power. Krugman (1998) recalled the concept of liquidity trap during the Japanese crisis, who stated that the liquidity trap is connected with a problem of credibility of the monetary policy, since the economic agents do not believe that the central bank is able to sustain the monetary expansion in the future.

In a situation of liquidity trap, the central bank needs to recur to unconventional monetary policies that include three measures which aim is to improve financing conditions beyond short-term interbank interest rates:

1. Influencing medium to long-term interest rate expectations
2. Expanding the size of central bank’s balance sheet
3. Changing composition of central bank’s balance sheet

The first measure is called also “forward guidance”, an interest rate policy that does not affect central bank’s balance sheet. This is a communicative instrument utilized by the central bank to signal in a transparent way the future actions of the central bank. The objective of forward guidance is to influence short, medium and long-term expectations of key interest rates. Recalling Campbell et al. (2012), we can classify two types of forward guidance: Odyssean and Delphic forward guidance. In the Odyssean type, the central bank explicitly says that as soon the economy will recover, it will not raise the
interest rates. In this way, the central bank binds itself to move interest rates also in case the GDP or inflation turn positive. In the Delphic type, the central bank implicitly asserts about key interest rate shifts depending on forecasts about GDP and inflation. When the central bank faces the ZLB problem, it can still influence expectations on interest rates, for example by committing to maintain policy rate at the lower bound for a certain period (Odyssean forward guidance). This process, through the term structure, will flatten the yield curve through lowering long-term interest rates (e.g. long-term government bond rates), therefore boosting lending activity, investment and consumption.

The second measure is the “quantitative easing”, a balance sheet policy that can be distinguished between direct and indirect, as Bini Smaghi (2009) asserts. Through the direct quantitative easing, the central bank changes the size of its balance sheet through the purchase of assets such as short and long-term government bonds. This increase of the assets’ side of the balance sheet is reflected with an increase of bank reserves in the liability side. On one side, the aim of direct quantitative easing is to reduce long-term interest rate of low risk assets in order to lower also other risky activity assets through market risk-adjustment mechanism. On the other side, the central bank injects more liquidity into banks’ balance sheets in order to make new loans; however, banks may decide to keep the new-created liquidity into their balance sheets, without influencing the real economy. Through the indirect quantitative easing, the central bank lends money to banks in exchange of low-risk assets, such as government bonds, as a collateral. Therefore, the central bank does not assume interest rate risk, market risk and sovereign risk into its balance sheet. This policy increases the monetary base in the system endogenously, depending on the state of financial stress of the banking sector.

The third measure is the “credit easing”, a balance sheet policy that can be distinguished between direct and indirect, as well as quantitative easing, as Bini Smaghi (2009) asserts. Through the direct credit easing, the central bank changes the composition of its balance sheet through the sale of less risky assets such as short-term bonds and the purchase of more risky asset classes such as ABS. Thus, the central bank assumes also credit risk into its balance sheet, leading policy-makers to evaluate carefully all the assets the central bank will be buying. Through the indirect credit easing, the central bank lends money to banks in exchange of risky assets as a collateral, thus not transferring the credit risk into the central bank’s balance sheet. The ratio of the indirect
Credit easing is similar to the indirect quantitative easing one, which is limiting the exposure of the central bank to the different risks. Both direct and indirect credit easing ease financing conditions from banks to the impaired sectors. Central bank’s aim is to reduce spreads between interest rates of assets classes which market is seriously impaired and of risk-free assets. These measures does not require central bank to increase its balance sheet size through, for example, increasing its bank reserves.

However, central bank can still use these unconventional monetary policies when interest rates are above zero, especially if the conventional monetary policy transmission process has become impaired in a very short period. This often happens during serious financial crisis, when banks stop making loans, financial asset prices drop and all the system falls in a liquidity trap, where every economic agent hoards liquidity. Anyway, it is important to keep in mind that the quantitative easing should be used only at the zero-lower bound condition in order to discourage banks to keep reserves in the central bank instead of using them to make loans. Contrary, the central bank can pursue the credit easing also in normal conditions, when the economy is not constricted in the zero-lower bound scenario.

### 1.3 Balance sheet policies

#### Figure 1.2: Balance sheet policies

<table>
<thead>
<tr>
<th>Market targeted</th>
<th>Impact on private sector balance sheets</th>
</tr>
</thead>
<tbody>
<tr>
<td>Foreign exchange</td>
<td>★ Change in net FX exposures, change in the composition of claims on the public sector</td>
</tr>
<tr>
<td>Public debt/securities</td>
<td>□</td>
</tr>
<tr>
<td>Private credit/securities</td>
<td>◆</td>
</tr>
<tr>
<td>Bank reserves</td>
<td></td>
</tr>
</tbody>
</table>

Exchange rate policy (★); Quasi-debt management policy (□); Credit policy (◆); Bank reserves policy (shaded area)

Source: Borio and Disyatat (2009)

To understand better both direct quantitative and credit easing, we need to analyze the basis form of balance sheet policy. As we can see in the figure (1.2), Borio and Disyatat (2009) classify the various types of policy based on two criteria:
1. Impact on the structure of private sector balance sheets
2. Market segment targeted

Regarding the impact on the structure of private sector balance sheets, we can distinguish three different impacts:

1. Impact on the net foreign exchange exposures
2. Impact on the composition of claims on the public sector
3. Impact on the composition of claims on public vs. private sector and on the profile of claims on private sector

Regarding the market segment targeted, we can distinguish four main areas:

1. Foreign exchange area
2. Public debt/securities area
3. Private credit/securities area
4. Bank reserves area

Connecting these criteria, we can distinguish four broad categories of balance sheet policy:

1. Exchange rate policy
2. Quasi-debt management policy
3. Credit policy
4. Bank reserves policy

In the exchange rate policy, the central bank recours to operations in the foreign exchange market in order to affect the exchange rate, altering the net exposure of the private sector to foreign currencies.

In the quasi-debt management policy, the central bank targets to the public debt’s market in order to alter the yield on government securities, influencing the cost of funding and asset prices. In doing this the central banks alters the composition of claims on the public sector held by the private sector.

In the credit policy, the central banks targets specific segment of the private debt and securities market, altering the composition of private sector balance sheet by changing the central bank’s exposure profile to private sector claims. Central bank can do this modifying the amount of private sector claims in his balance sheet or modifying the
composition between private and public sector claims held by the private sector. We can distinguish two credit policies:

1. Credit policy that influences interbank market conditions (e.g. long-term operations, inter-central bank FX swap lines, broadening of eligible collateral, etc.)
2. Credit policy that influences nonbank credit market (e.g. commercial paper purchase, ABS purchase, corporate bond purchase, etc.)

In the bank reserves policy, the central bank sets a specific target for bank reserves in the liability side of its balance sheet, regardless of how this is counterbalanced on the asset side. It is impossible to know the impact on private sector balance sheets since it depends on the asset counterpart to the reserves’ expansion. How Borio and Disyatat (2009) assert, this policy is extremely important since bank reserves are the only acceptable mean to achieve final settlement of all transaction and they can be hold exclusively by banks. Supplying bank reserves in period of financial crisis improves financial stability among banks since they recur a lot less to the interbank market respect to normal economic times.

It is important to notice that balance sheet policies expose central bank to different risks:

1. Quasi-debt management policy exposes central bank to the interest rate risk, market risk and sovereign risk
2. Credit policy exposes central bank also to varying degrees of credit risk, which become acute during financial crisis

Referring to the basic balance sheet policies, direct quantitative easing can be seen as a mixture of bank reserves policy and quasi-debt management policy, together with a specific communication strategy about the future, as pointed out by Borio and Disyatat (2009).

Ugai (2007) defines three features of quantitative easing:

1. Explicit target for bank reserves
2. Conditional commitment to maintain high reserves levels into the future
3. Increased purchases of government bonds to facilitate the attainment of the target of bank reserves
Direct credit easing can be seen a mixture of quasi-debt management policy and credit policy that influences nonbank credit market, since it reassembles central bank balance sheet, shifting from assets of certain risk to other of different risk, in order to support impaired credit markets.

Lastly, the indirect credit/quantitative easing can be seen as a mixture of quasi-debt management policy, credit policy that influences the interbank market conditions, and bank reserves policy.

Although there is still much discussion about the transmission mechanism of the unconventional monetary policy, Joyce et al. (2011) list five main channel through which balance sheet policies can transmit their effects (figure 1.3):

1. Signaling channel
2. Portfolio rebalancing channel
3. Liquidity premia channel
4. Bank lending channel
5. Confidence channel

**Figure 1.3: Unconventional monetary policy transmission channels**
The signaling channel regards the communication of balance sheet policy by the central bank about future course of the policy. For example, asset purchases may lead financial markets to expect policy rates to remain low for a certain period to meet the inflation target. Krishnamurthy and Vissing-Jorgensen (2011) assert that the signaling channel affects all bond market interest rates across the yield curve with effects depending on bond maturities. They also find out that the signaling channel should have a larger impact on intermediate-maturity than on long-maturity rates, since the commitment to keep rates low lasts only until the economy recovers and the central bank can sell the accumulated assets.

The portfolio-rebalancing channel regards the changes made by the private sector in their portfolio. By purchasing a large quantity of assets held by the private sector, central bank change the relative supply of the assets. Since the base money issued by the central bank and the financial assets purchased are not perfect substitutes, the private sector, which previously sold those assets to the central bank, will buy other assets which have similar characteristics to the assets sold. On this channel, Curdia and Woodford (2010) assert that the degree of substitutability depends on credit imperfections and heterogeneity, factors that support credit easing policy and not QE policy. Indeed, the presence of imperfections in private financial intermediation and the possibility of disruptions to the efficiency of financial intermediation through banks leads the central bank to utilize credit easing to help private sector to get rid of those impaired assets. Private sector could then use the newly created money into other investments, thus boosting output and inflation. For Curdia and Woodford (2010), the reason why QE is ineffective is because government bonds pay a safe rate identical to rate set by central banks and the result is that bank reserves and government bonds become perfect substitutes. Anyway, in their study this result depends on the fact that government bonds which central bank buys are short-lived assets with identical characteristics to bank reserves. To generate an impact the QE has to influence investors’ portfolios and central bank has to buy long-term government bonds, which are less liquid than the short-term ones. Doing so, central bank forces private investors to find similar assets to long-term government bonds in terms of liquidity and not only, pushing up also other assets. This process, therefore, pushes up both assets involved in the balance sheet policy and close substitutes assets to the claims bought. Borio and Zhu (2008) assert that easier funding conditions or the removal of risky assets from portfolios may reduce perceived risks and induce higher risk-taking, the so known
“search for yield”, widespread during QEs policies. Higher asset prices cause lower yields, so lower borrowing costs for firms and households. In same time, it should stimulate spending by increasing wealth of asset holders. Summarizing, we can consider the portfolio-rebalancing channel as direct channel of balance sheet policies, since it affects private sector’s portfolios and so spreads and investments.

The liquidity premia channel regards the increase of liquidity in the hands of the private sector. Joyce et al. (2011) assert that increased liquidity and improved market functioning will lower premium of illiquidity and therefore increase asset prices, easing credit conditions for companies that are more stimulated to raise funds.

The bank-lending channel regards the lending activity of the banks. When central bank conducts asset purchases, the banking sector generates new reserves at the central bank. This increase of liquid assets should lead banks to make new loans, instead of what they have would done without central bank intervention. Joyce and Spaltro (2014) assert that QE policy made in UK during the crisis may have led to an increase in bank lending through this channel. They find evidence that bank lending is positively related to how well capitalized banks are, suggesting that the impact of QE on bank lending may have been weaker because of lower levels of capital during the crisis.

Lastly, balance sheet policies should also improve economic outlook and therefore have broader confidence effects. The boosted confidence, on one hand, should encourage investment and consumption, and on other hand should increase further asset prices by reducing risk premium.

Theoretically, these channels should contribute to boost asset prices, to increase total wealth and to lower cost of borrowing for the private sector. If households consume part of the increased wealth, companies invest some of the extra funding raised on capital markets, or banks start again to make loans, demand will be higher. In last instance bank should achieve an increase of spending and income and so obtaining the inflation target and the real economic growth.

Comparing the two transmission mechanisms showed, we could say that the conventional monetary policy transmission mechanism works through the control of the key interest rates, which in turn influence money market interest rates, credit supply, asset prices and foreign exchange rates. Therefore, the central bank affects the yield directly through the shifts of the short-term rates, which in turn affect medium and long-
term rates. Instead, with unconventional monetary policy the central bank affects the yield curve more indirectly, influencing medium to long-term interest rates by purchasing different asset prices. Thus, the central bank reduces term spreads between short and long-term rates and credit spreads between risk-free assets and risky assets, influencing wealth, cost of borrowing, spending and income.

1.4 Direct quantitative easing in theory
As mentioned before in the paragraph (1.2), direct quantitative easing is a mix of bank reserves policy and quasi-debt management policy. On one hand, the central bank increases monetary base through increasing bank reserves, on the other it buys huge amount of government bonds. The increase of bank reserves is justified because it help banks’ stability, since they recur much less to the interbank market. The purchase of government bonds is justified by lowering the yields on purchased bonds, on bonds of similar risk and partly on corporate bonds through the “search for yield” of market agents. This effect would stimulate longer-term investments and aggregate demand through the transmission mechanism.

Theoretically, on one side, the additional liquidity supplied to banks through bank reserves should be used to extend new credit. Of course, there is the risk that banks choose to hold those reserves at the central bank because they do not find any good opportunity to lend this money to the real economy. In fact, during periods of financial crisis it is common to see deleveraging of bank activities, since banks become more adverse to risks. In this case, money remains limited in the financial sector. How Bini Smaghi (2009) asserts, this risk can be minimized if the central bank conducts this type of operation only at the lower bound. Indeed, in this scenario banks have no remuneration by holding bank reserves at central bank. That is the reason why QE should be implemented only at lower bound. In fact, Borio and Disyatat (2009) assert that the amount of credit outstanding is determined by banks’ willingness to supply loans, based on the perceived risk-return trade-offs, and by the demand for those loans, and not by the amount of reserves supplied.

On the other side, Borio and Disyatat (2009) assert that what is really boosting real economy macro-variables is the flattening of yield curve generated by quasi-debt management operation through the purchase of long-term government bonds. Central bank, doing so, also accommodates fiscal policy of government authority through very low interest rate. Government authority will have then more money to increase public
spending and so aggregate demand through Keynesian multiplier. Milton Friedman agrees on this vision; in fact, at a conference in 2000, he said (as quoted by Beckworth, 2011): “Now the Bank of Japan’s argument is, “Oh well, we’ve got the interest rate down to zero; what more we can do?” It’s very simple. They can buy long-term government securities, and they can keep buying them and providing high-powered money until the high-powered money starts getting the economy in an expansion”. Anyway, Woodford (2012) asserts that a policy of creating additional reserves in order to purchase long-term treasury securities does not constitute quantitative easing in the “pure” term. Pure quantitative easing, which should not be confused with the direct quantitative easing concept, is characterized by the creation of bank reserves through the purchase of short-term government bonds only. Thus, general quantitative easing can be viewed as the composition of two policies:

1. Pure QE in which new reserves are created by purchasing short-term government bonds
2. Operation Twist in which central bank sells short-term government bonds to buy long-term government bonds

Recalling Blinder (2010), the quantitative easing is able to lower long-term interest rates through shrinking both term premiums and risk and liquidity spreads. Term premiums usually go down thanks to the arbitrage on the yield curve, through which long-term interest rates reflect short-term interest rates. Shrinking risk and liquidity spreads on government bonds, it reduces also the interest rates on loans, even if riskless rates are unchanged.

We need to keep in mind that the effectiveness of the quantitative easing depends on the degree of substitutability across the assets being traded. In fact if the central bank, with supply of bank reserves, buys short-term government bonds, it does not help the economy since it cannot flatten the yield curve since bank reserves and short-term government bonds, in a zero-lower bound situation, are almost perfect substitutes. Indeed less the assets are perfect substitutes and more effective is the portfolio-rebalancing channel of the transmission mechanism.

1.5 Direct credit easing in theory
Direct credit easing is a policy that consists in the purchase of commercial paper, corporate bonds and asset-backed securities in order to reduce liquidity shortages and
credit spreads in certain credit markets. This policy reduces the supply for private investors, increasing asset prices and therefore reducing yields. Hence, credit easing has higher impact if spread between government bonds’ yield and commercial papers and other securities of this type is higher, because the two assets are not perfect substitutes, affecting primarily the portfolio-rebalancing channel. Moreover, credit easing has a higher impact on the bank-lending channel than the quantitative easing, since it can relax balance sheet constraints with more ease.

In this case, central bank interacts directly with the private sector also taking the credit risk on its balance sheet, which can compromise its financial independence. Direct credit easing, however, does not change the size of central bank’s balance sheet and so does not have an impact on the supply of monetary base. As Bini Smaghi points out (2009), policy-makers need to assess the eligibility of all assets because of the implications they could have for the credit risk exposure of the central bank’s balance sheet. Another problem that policy-makers need to face is a good planning of purchase program in order to avoid allocative distortions in terms of industries. Finally, central bank needs to find proper exit strategy from this unconventional monetary policy, since the assets purchased have higher credit risk and are less liquid than government bonds.

1.6 Indirect quantitative/credit easing in theory
Contrary to the direct quantitative and credit easing, in the indirect ones the central bank does not directly hold the assets and it assumes no risk on its balance sheet. The central bank lends to bank at longer maturities than normal OMOs against a collateral, which includes also assets whose markets are temporarily impaired, as explained by Bini Smaghi (2009). This policy affects directly the yield curve over the period at which those operations are conducted. In other words, longer period the central bank conducts this policy and bigger part of the yield curve is affected. In the indirect quantitative and credit easing processes, the increase in the monetary base is determined endogenously by the banking system, since banks decide whether or not to get liquidity from central bank in exchange of collateral.

Through these operations, during periods of financial stress, the size of central bank balance sheet increases since banks have an increased demand of excess reserves. Enlarging the pool of collateral accepted for refinancing operations, banks would deposit a great amount of asset of lower quality, helping those sectors that are currently impaired.
Anyway, in this case central bank has no risk on its balance sheet, since the amount of collateral will vary endogenously depending on the financial stress of the banks. In fact, when banks will restore their equilibrium, the amount of deposited collaterals of this type should become lower.

1.7 Empirical evidence from Japan
While in the paragraph (1.4) I explained in theory the main characteristics of the direct and quantitative easing, this paragraph shows now an empirical evidence of this monetary policy. One example of direct quantitative easing is the one that Bank of Japan did from 2001 to 2006.

During 1990s, Japan’s CPI gradually reduced until becoming negative after burst of IT bubble in 2000. In 2001, Bank of Japan undertook quantitative easing in order to block price decline and prepare the basis for new economic growth. The unconventional monetary policy took place at the lower-bound policy and consisted of four pillars:

1. Make the commitment that the above ample liquidity provision would continue to stay in place until the CPI registered stably at zero percent or an increase year on year
2. Change the main target for money market operations from uncollateralized overnight call rate to the outstanding current account balances held by financial institutions at the BOJ and provide ample liquidity to realize a CAB target substantially in excess of the required reserves
3. Increase the amount of outright purchases of long-term Japanese government bonds
4. No commit to maintain the increased supply of reserves permanently

While the first point is a clear reference to the forward guidance, the second and third points regard quantitative easing. While point two regards the liability side of the central bank’s balance sheet, the third one regards its asset side.

Bank of Japan started increasing its balance sheet’s liabilities, not caring about nature of assets acquired with new created money. Therefore, for the bank it was most prudent to buy mainly government bonds.

The QE started with a current account balance (CAB) target of 5 trillion yen (about 1% of GDP). After, the target was progressively increased in response to the decline in
economic activity. The target reached finally 30-35 trillion yen (about 7% of GDP) in January 2004 and remained unchanged until March 2006.

Looking at the figure (1.4), we can see that Bank of Japan balance sheet jumped from 80 trillion yen in 2001 to 150 trillion yen in 2006. On the asset side, this increase was mainly due to the purchase of bills and government securities, which was reflected on the liability side with an increase of current deposits and repo operations.

Figure 1.4: Bank of Japan balance sheet
Regarding the term spreads between the TIBOR (Tokyo interbank offered rate) and the overnight call rate, the figure (1.5) shows that they were shifting between 80 basis points and 10 basis points during the ZIRP (zero interest-rate policy) period. During QE period, the term spreads declined towards 10 basis points and remained stable until 2006. This process produced the significant effect of mitigating liquidity risk in money markets.
On the other side, the QE influenced credit spreads (figure 1.6). The credit spread for banks between the rate on certificates of deposit and the rate on three-month treasury bills decreased towards 10 basis points after the introduction of the QE. Instead, the credit spread for non-banks activities between the credit product indicators across ratings and the three-month Treasury bill rate declined by 150 basis points but with a certain time lag. This reduction helped to lower the cost of external finance of those businesses. Anyway we need to keep in mind that BOJ achieved these credit spreads reduction without intervening heavily in credit markets like Fed did some years after, reminding that the US financial scenario was a lot worse than Japan’s one.
However, Japan’s QE produced certain side effects on the uncollateralized call market. In fact, as figure (1.7) shows, there was a huge decline in the outstanding amounts in that market which went from 20 trillion yen in early 2001 to 3.4 trillion yen in December 2002. This decline did not significantly reverse in 2004-05 even after Japan’s financial system restored its stability, by resolving the nonperforming-loan problem, as Shiratsuka (2010) asserts. This happened because under QE, banks lost the incentive to engage transactions in the call market and lenders barely covered transaction costs, since the rate was very close to zero. Borrowers instead did not need to raise funds in the call market because BOJ was the main source of financing.
Indeed, in the figure (1.8) we can clearly observe that bank loans were dropping during the QE period. In this context, as Bini Smaghi pointed out in 2009, given the high degree of de-leveraging which the Japanese economy, and the banking sector in particular, was undergoing, banks did not find themselves in a position to pass on the additional liquidity to the non-financial sector.

As the Bank of Japan stated in its annual review of 2006, the Japanese economy was continuing to recover. Both domestic and internal demand were increasing, pushing up the production. As supply and demand conditions continued to improve, the change in consumer prices turned positive from November 2005. In March 2006 CPI was growing
at a rate of 0.5% and the central bank stated that this growth was expected to remain positive. Since the QE succeeded to bring inflation above zero, the BOJ exited the quantitative easing and decided to change monetary policy target going back to the uncollateralized overnight call rate. During the months after March 2006 Bank of Japan reduced CABs in order to rise, in July 2006, the policy rate for the first time.

Regarding the impact on wider economy, the empirical evidence suggests that the expansion of the central bank’s balance sheet had limited effects on the real economy. In the figure (1.9), there are listed three econometric studies on the CPI, industrial production and GDP. The studies assert that before the 2000 QE policies have been effective at influencing the real economy, in terms of CPI, industrial production and GDP. Regarding the 2001-06 QE, some studies show that it had a slightly positive impact on the real economy variables, while others show that had no impact or even negative impact.

**Figure 1.9: Empirical surveys**

<table>
<thead>
<tr>
<th>Variables</th>
<th>Period</th>
<th>Effects on prices</th>
<th>Effects on real economy</th>
</tr>
</thead>
<tbody>
<tr>
<td>Kimura et al.</td>
<td>1985</td>
<td>CPI rate of change: yes (positive)</td>
<td>GDP gap: uncertain (slightly positive)</td>
</tr>
<tr>
<td>2002</td>
<td></td>
<td>CPI rate of change: no</td>
<td>GDP gap: no</td>
</tr>
<tr>
<td>(2003) Bayesian VAR</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Markov Switching</td>
<td>After 1998</td>
<td>CPI: insignificant (close to zero)</td>
<td>Industrial-production: insignificant (slightly positive)</td>
</tr>
<tr>
<td>VAR</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>4</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Correction Model</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>6</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Source: Ugai (2007)

Analyzing these results, Ugai (2007) concludes in his survey on empirical studies on the effects of QE that the consequences of expanding monetary base are generally smaller than that coming from the forward guidance policy.
As Shiratsuka (2010) asserts, the QE played a certain role in bolstering Japan’s economy, in particular by stabilizing financial system. Such stimulating effects failed to be transmitted outside the financial system, however, suggesting that the transmission channel between the financial and nonfinancial sectors has been blocked.

Haltmaier, Martin and Gust (2008) estimate that the QE helped to reduce long-term interest rates by about 50 basis points, which might have raised GDP growth and inflation between 0.50% and 0.75% year on year by boosting aggregate consumption and investment and boosting net exports thanks to a fall of foreign exchange rate.

However, Bini Smaghi (2009) pointed out that if the quantitative easing had last longer it actually could have had an expansionary effect through easing the fiscal policy. Doing so, the central bank could have increased aggregate demand in three ways:

1. By facilitating the issuance of government bonds
2. By affecting the risk premium of the yield curve
3. By re-anchoring inflation expectations to a positive target

Of course, these effects depend on how much time quantitative easing will last for and on credible commitment to a well-defined exit strategy, since central bank assumes particular risks in undertake these types of policy.

1.8 Empirical evidence from the US
Bernanke (2009) has described the Federal Reserve’s approach in recent US 2008-2010 experience as a credit easing policy, even though the Fed actually increased its balance sheet size, making this policy resemble the Japan’s quantitative easing. The reason relies in the fact that Fed approach focused more on the assets side of the balance sheet rather than the liability side. In fact, the US central bank preferred to develop lending facilities for the impaired credit markets and to purchase impaired credit assets, such as GSE and MBS, instead of buying long-term government bonds (thing that pursued only 2 years after). Recalling the paragraphs (1.5) and (1.6), where the main characteristics of direct and indirect credit easing are explained in theory, this paragraph explores these in practice.

In 2008, the Fed has established different lending and purchase facilities in order to provide liquidity and to improve the functioning of impaired credit markets. First set of policy tools is characterized by two main lending facilities:
1. Term auction facilities (TAF) which helps financial institutions to secure an adequate access to short-term credit, with refinancing operations based on fixed amount communicated in advance from central bank each month where the interest rate is fixed through competitive auction

2. Commercial paper fund facilities (CPFF) which provides to increase the liquidity in the short-term funding markets through a special purpose vehicle (SPV) that purchases three-month unsecured and asset-backed commercial paper from eligible issuers

However, as Bernanke (2009) pointed out, the provision of credit to financial institutions exposes the Federal Reserve to only minimal credit risk since the loans that were made to banks and primary dealers through the facilities were over-collateralized. Fed has never suffered any losses in the course of its normal lending to banks.

Moreover, the Federal Reserve has developed a secondary set of policy tools, which involve the provision of liquidity directly to borrowers and investors in key credit markets, for example facilities:

- To purchase highly rated commercial paper at a term of three months
- To provide backup liquidity for money market mutual funds

Moreover it has been established a facility which lends against AAA ABS collateralized by students loans, auto loans, credit card loans and loans guaranteed by the small business administration. In addition, in this case the credit risk exposure of the Fed is minimal, thanks to collaterals and Treasury partial coverage, as Bernanke announced in 2009.

Third set of policy tools constituted to support the functioning of credit markets involves the purchase of longer-term securities for the Fed’s portfolio, especially government-sponsored enterprise debt (GSE) and mortgage-backed securities (MBS), which is known as QE1 or LSAP I. The first operation has the aim to enhance the availability and to reduce the cost of credit to the targeted borrowing sectors primarily by reducing the risk of capital losses to investors. The second operation has, instead, the aim to reduce mortgage rates in order to recover real-estate market hit by the crisis. This third set of policy tools carries more credit risk exposure than the other two, but it also remunerates the Federal Reserve with higher yield.
The first two policy tools are part of indirect credit easing policy, since they work through collateralization process, so minimizing central bank liquidity and credit risks. Instead, last policy tool is part of direct credit easing, since central bank buys those assets and keeps them in its balance sheet. These assets will not change endogenously like the first ones, since it is up to the central bank to choose the moment to sell those assets.

As shown in the figure (1.10), the focus of the Fed has been on credit programs and mortgage-backed securities, in order to help impaired credit markets, support loans and sustain financial stability of banks and financial system in general. Total assets skyrocketed from $907 billion on September 3, 2008 to $2.214 trillion on November 12, 2008. On the liabilities side, bank reserves ballooned from about $11 billion to $594 billion in the same period. Furthermore, we notice that the Fed, between July 2008 and March 2009, diminished the quantity of US treasury securities in its portfolio to increase credit facilities; at the end of 2009, Fed increased again the quantity of US treasuries in view of the QE2 policy.

Figure 1.10: Fed balance sheet

![Fed balance sheet](image)
Regarding the term spreads between the three-month LIBOR and the overnight rate, the figure (1.11) shows that they reached a peak of 350 basis points in late 2008, signaling a very high liquidity risk in the money markets. The reduction of short-term interest rates to zero and the asset purchases pursued by the Fed helped to lower the liquidity risk in these markets, thus lowering the term spreads towards 20 basis points.

**Figure 1.11: Term spreads**

![Graph showing term spreads between 2007 and 2011](image)

Source: British Bankers’ association and Prebon

Conversely, Fed operations helped to reduce credit spreads between different rated asset classes and the 10-year Treasury bonds as well (figure 1.12). Contrary to the Japanese experience, in this case the credit spreads fell almost instantly as the Fed introduced these measures. However, it is important to recall that the Fed, differently from the BOJ, pursued a more aggressive policy, intervening directly and heavily on the credit markets.
Fed intervention had a strong positive impact on mortgage rates, as shown in the figure (1.13). We can classify two types of mortgage rates: conforming and jumbo rates. While conforming rates are related to mortgages loans that conform to GSE, which are loans that respect some value limits, jumbo rates are related to mortgage loans that do not conform to GSE and do not respect those limits and so they are more risky. In the figure we can see that jumbo and conforming rates decreased respectively 200 and 150 basis points between 2008 and 2010, signaling a recovery of confidence in the mortgage market.
Regarding the interest rates in general, Gagnon et al. (2011) concluded that the LSAP I had significant effects on longer term interest rates on a variety of securities, including treasuries, agency MBS and corporate bonds. They estimated that Fed’s measures reduced 10-year term premium by something between 30-100 bps.

What about the impact on the wider economy, in the annual report of 2010, the Federal Reserve asserted that the economy activity was expanding at a moderate pace in the first half of 2010. Although GDP and inflation were gradually recovering, employment remained significantly below pre-recession levels. Moreover, bank credit was still tight for many borrowers. Therefore, since the overall economic situation was not totally recovered, Fed decided to continue with another round of QE in 2010, the LSAP II, where it purchased about $600 billion of long-term government bonds.

What about empirical studies on the real effects of LSAP programs, Baumeister and Benati (2010) estimated, through a structural VAR, that the asset purchases led to compression in the long-term yield spread, exerting a powerful effect on both output growth and inflation. Also Chung et al. (2012) studied the impact of the Fed’s LSAP I and II (considering both CE and QE policies), and they found out that the Federal Reserve helped in raising level of real GDP by 3% and that the inflation is 1% higher than in the case US central bank had not carried out the programme of purchases, implying that asset purchases prevented deflation spiral.

1.9 Empirical evidence from the EU
In contrast with the previous examples, the 2008-2013 ECB monetary policy cannot be defined as direct quantitative easing, neither direct credit easing, since the ECB did not take risk exposures to the public sector through government bonds’ purchases or to the private sector through MBS or other credit instruments’ purchases. Thus, ECB policy has been described as an indirect credit/quantitative easing by Bini Smaghi (2009), which the aim to to relax banks’ collaterals and funding liquidity constraints, in order to extend new loans and expand credit supply.

ECB interveined in May 2009, reducing the main refinancing rate to 1% and introducing 12-months LTROs (long-term refinancing operations) and covered bond purchase program (CBPP). 12-months LTROs have the aim to support bank lending and money market activity, addressing banks to borrow at longer maturities, while CBPP has the aim to support the covered bond market; this latter was seriously impaired after
the Lehman Brothers bankruptcy. How Fawley and Neely assert (2013), this market is extremely important for the banks since they can allievate the maturity mismatch they face when making long-term loans, while having short-term deposits on the liability side. Both these operations were collateralized by banks, thus reducing to minimum the risks ECB was undertaking. Furthermore, ECB intervened again in the end of 2011, when it announced the two three-year LTROs and the Securities Market Programme. This latter consists in the intervention of the ECB in public and private debt securities markets in order to restore the monetary policy tranmission mechanism in dysfunctional market segments. The liquidity generated through these operations is consequently reabsorbed in order to do not affect the ECB monetary policy stance of price stability in the medium term.

**Figure 1.14: ECB balance sheet**

However, as we can see in the figure (1.14), the main sources of the ECB balance sheet’s expansion between 2008 and 2013 were the long term refinancing operations (LTRO). The first LTRO measures had maturities between six and twelve months, which were created to let the banks borrow at longer maturities than the normal OMOs. After the ECB undertook three-yar LTROs, through which the balance sheet expansion reached its peak:
• 1<sup>st</sup> three-year LTRO occurred on 22 December 2011, when 523 banks participated demanding for €489 billions
• 2<sup>nd</sup> three-year LTRO occurred on 29 February 2012, when 800 banks participated demanding for €529 billions

In exchange of these loans the ECB required different collaterals, including very low-quality assets such as Greek government bonds. In fact, since 2008 the ECB has constantly expanded the list eligible assets as collateral for those type of operations in order to help the financing process of the banks. Moreover, since many assets were downgraded from 2008 onwards, the central bank has been forced to accept more and more low-rated collateral.

As the figure (1.15) shows, the spread between the Euribor and the EONIA, a key factor to evaluate the health of the European interbank market, reached its peak during December 2011. After the announcement of LTROs by the ECB, this spread started to reduce, going towards 10 basis points, signalling a recovery of the interbank market.

**Figure 1.15: Term Spreads**

![Euribor EONIA spread](source)

Source: Thomson Reuters Datastream

Last set of measures undertaken by ECB were the OMT (outright monetary transactions). Those operations were announced in July 2012 and consisted in the purchase of short-term government bonds of countries in macroeconomic difficulty. Also in this case, the liquidity injected into the market has been completely sterilized in order to control the inflation rate.
As the figure (1.16) shows, the two three-year LTRO, together with several announcements of ECB and OMT measures helped to reduce government bond spreads with the german government bonds.

**Figure 1.16: Term Spreads**

![Graph showing term spreads](source: Thomson Reuters)

However, those measures were not enough to stimulate the real economy. In fact, although the euro area GDP growth rate turned positive in 2013 but it remained near zero (figure 1.17), the inflation was decreasing steadily, turning negative in the end of 2014 (figure 1.18). This led the ECB to pursue more aggressive unconventional monetary policies such as quantitative easing, which planned to purchase €60 billion government bonds per month for at least 18 months.
1.10 Central bank risks
When the central bank recours to unconventional monetary policies such as direct quantitative easing and credit easing, its balance sheet becomes more exposed to market developments, for example a future rise of long-term interest rates could reduce the value of central bank assets. Thus, the capital of central bank can be at risk and this can undermine the central bank’s credibility. While the purchase of long-term debt involves duration and hence market risk, credit policies, in addition, generate credit risk exposures. If central bank fails to push down interest rates or if these will start to grow
after some time the start of unconventional policy, maybe thanks to increase of expected inflation and so on nominal interest rates, central bank may lose its financial independence and its operational autonomy. How Borio and Disyatat (2009) assert, the loss of operational autonomy could be especially dangerous in an economy that emerges from a protracted period of financial strains and depressed activity. Also in the indirect policies the central bank exposes to the risk that the collaterals value will decrease and the counterpart will default, but these effects can be mitigated through a daily revaluation of those collaterals and through the acceptance of counterparties with a good financial position.

Moreover, the central bank needs to take care about the law of the diminishing returns in the quasi-debt management policies. In fact when central bank for the first time announces this type of monetary policy, it has a strong impact on long-term yields. Anyway if the central bank announces additional steps of a similar size monetary policies, there is no guarantee that this will have equivalent results. Anyway, the central banks needs to take care if markets become excessively concerned about the potential inflationary implications of these policies, undermining their effectiveness, as Borio and Disyatat (2009) pointed out. Regarding the credit policies, there is the risk that markets may become too much dependent on central bank support. As told before, policy interventions in this case may distort the level playing field between those who receive the help and those who do not receive it.

Another set of risks that central bank needs to care about regard the exit strategies. In fact, while entering in an unconventional monetary regime is not so difficult, the real problems stay beyond it, when the central bank has to restore the normal financial and economic conditions, returning to a interest rate policy. In this context central bank can’t exit too fast from policies of QE or credit easing because it can hamper the recovery. Opposite problems happen if central bank exit too slow from those policies, creating a new set of financial imbalances or lead to inflationary pressures, how Borio and Disyatat (2009) assert. Anyway everyone agrees to the fact that the central bank must sell every newly bought assets before it starts to increase interest rates, in order to do not suffer big losses on those assets and do not hamper the economic relief. The central bank should minimize any negative effects by announcing sale programs for the next six years, for example.
2. Methods of evaluation of unconventional monetary policies
This chapter explores the functioning of DSGE models and their role in the monetary policy evaluation. The paragraphs (2.1-2.3) analyze the general traits and the evolution of DSGE modeling, showing the ratio behind those models and their basic structure. The paragraph (2.4) distinguishes between different classes of elaborated DSGE models for monetary policy evaluation. The paragraph (2.5) explores an elaborated DSGE model developed in order to isolate the portfolio-rebalancing channel of the balance sheet policies. The paragraph (2.6) shows the functioning mechanism of this model and the main results. The paragraph (2.7) explores the relationship of this transmission channel with the past quantitative and credit easing experiences. Finally, the paragraph (2.8) extends the analysis on a wider range of elaborated DSGE models, which can capture also other transmission channels; lastly, it explores some limits of DSGEs.

2.1 From Keynesian theory to DSGE modeling
The empirical studies around the QE have stressed different economic models, from econometric models to DSGE (dynamic stochastic general equilibrium) models. The latter ones are the result of a long development process around economic models, which started with the Great Depression.

How Snowdon and Vane (2005) assert, the Great Depression gave the stimulus to many economists to develop new models in order to explain the real phenomena, which led to the birth of modern macroeconomics. With Keynes, we have the birth of modern macroeconomics as a systematic approach, which aggregates different phenomena, going from the real economy to money and interest rates. Contrary to the classical theory, Keynes (1936) asserted that capitalism had a fundamental flaw in the price mechanism, since the laissez-faire cannot allocate the resources in the best possible way, causing failure and involuntary unemployment. For Keynes, the employment and output are driven by the aggregate demand, characterized by consumption and investment. During downturns, the government should increase its spending in order to increase the aggregate demand, dampening the fall of employment and output. Thus, the intervention of the State is necessary to guarantee full employment, especially during downturn economic periods, when households reduce their consumption, slowing the economy even further.
The economists that followed Keynes, the neo-Keynesians, elaborated the neoclassical synthesis”, obtained through the combination of Keynesian macroeconomics with neoclassical microeconomics. John Hicks developed this theory, which was then elaborated mathematically by Paul Samuelson through the well-known IS-LM model. The synthesis is composed by the Keynesian theory in the short run and by the neoclassical theory in the long run. Thus, government and central banks can modify the short run equilibrium, knowing that the system will reach its equilibrium in the long run without any intervention. Regarding the unemployment, Phillips (1958) elaborated a curve that related unemployment with wage rate (and so the inflation rate). Phillips found out that those variables were inversely correlated and when the economy has zero percent growth in wage rates, the unemployment is about 5.5%. Thus, government and central bank face a trade-off when deciding the rate of inflation and unemployment they want to reach.

An important pillar that has been useful for the development of DSGE models has been the rational expectations theory of the new classical economics. This theory has been developed by Robert Lucas (1973), which replaced the adaptive expectations hypothesis which economists had been used until then. For him, the economic agents’ future predictions affects the value of an asset today and those predictions are based on the optimal forecasts and not on past events.

However, during the 1970s, how Gali and Gertler (2007) assert, economists and policymakers began to be skeptical about large-scale Keynesian models used for monetary policy evaluation, for two related reasons. First, some existing models failed to forecast the stagflation of the oil crisis during the 1970s. Second, two macroeconomists, Lucas and Sims, made two important critiques to the fundamentals underlying those models. Lucas (1976) argued the fact that the equations in those models were not structural, meaning that they were not policy-invariant. This means that, whenever policy changes, also these equations change and become unreliable. For Lucas, this problem was caused by the absence of an optimization process behind the developing of those equations. Similarly, Sims (1980) argued the fact that large-scale macro-econometric models did not identify endogenous and exogenous (predetermined) variables, which made the parameters estimated unstable across different policy regimes.
Meanwhile, Lucas (1973) started to elaborate business cycle models, which Kydland and Prescott (1982) refined and after developed real business cycle theory (RBC). The starting point of those models is the economic growth model of Solow-Swan. This classic model converges into a steady state in which the macroeconomic variables pro-capita are constant in the time without the presence of cyclical fluctuations. The RBC models are similar to the Solow-Swan model, but with the addiction of stochastic processes that cause technology shocks or, more in general, supply shocks. Therefore, these shocks develop the business cycles. The idea is that stochastic processes set up a series of intertemporal substitutions mechanisms regarding consumption, investment, labor supply, together with the law of capital accumulation, ensuring the fluctuation of macroeconomic variables around the steady state. Thus, the economy will reach new equilibrium through the adjustment of the demand given the shift of the supply curve, without any government intervention. Another characteristic of those model is the absence of frictions or imperfections and the irrelevance of money, thus the cycle is not money-induced.

In response to the critics made by Lucas (1976) and other new-classical economists, new-Keynesian economists adjusted some pillars of their theory. This new-Keynesian theory assumes that households and firms have own rational expectations and that prices and wages are sticky, meaning that they not automatically change to shifts of economics conditions. The reason relies, on one side, on the presence of “menu cost” meaning that some costs are necessary to adjust the nominal value of prices. On the other side, how Mankiw (1985) asserts, if one firm lowers own prices because of a fall of money supply, this will increase real income to the costumers, which will be able to buy more quantities of that good but not necessarily from the firm that lowered its prices. This mechanism leads firms to do not lower prices so fast in response to shifts of economics conditions. All this process makes difficult to reach the goal of achieving full employment. Thus, new-Keynesian economics supports the action of governments and central bank in order to achieve the macroeconomic stability, such as sustained growth and inflation, leading to a better result than the laissez faire. Another important assumption of new-Keynesian theory is the presence of monopolistic firms. This assumption is necessary because in a perfect competition regime the firms the price stickiness would not exist since firms would change price whenever they want in order to obtain the biggest slice of the market. In this way, firms apply a price higher than their marginal cost and achieve profits.
From the combination of RBC models and New-Keynesian models, economist developed DSGE models. Goodfriend and King (1997) use the term “New Neoclassical Synthesis” to describe this new type of models. As Dou et al. (2015) assert, the key innovation of the New Keynesian DSGE model for monetary policy evaluation is to incorporate nominal stickiness, agents’ behaviors, preferences of firms and households and the resulting monetary non-neutrality into a dynamic general equilibrium framework. The structure and the robustness of those models have pushed most central banks, like US Fed, ECB, BoE, to utilize DSGE models to make estimations about macroeconomic shocks and not only.

2.2 Reasons to use DSGE models and main characteristics
Dou et al. (2015) list different important reasons why central banks have increasingly been using DSGE models in their evaluations. First, the explicit role of expectations and its identification of deep structural parameters makes the DSGE models less subject to the Lucas (1976) critique, since those parameters are less likely to change to shifts in policy regimes. While the structural parameters are policy invariant, the reduced-form parameters depend on exogenously determined parameters set by public policy makers. The DSGE models are able to relate the reduced-form parameters to the deeper structural parameters. Second, DSGE models are less subject also to the Sims (1980) critique since they are able to distinguish endogenous vs. exogenous variables and endogenous vs. exogenous shocks.

DSGE models are general equilibrium models because they aim to describe the behavior of the economy as a whole, by analyzing the interaction of many microeconomic decisions, regarding consumption, saving, investment, labor, production, etc. made by households, firms, government and central bank in a macroeconomic framework. They are also dynamic models because they study the evolution of the economy over time. How Sbordone et al. (2010) assert, those models emphasize agents’ intertemporal choice in their behavior. The fact that current choices depend on future uncertain outcomes make the models dynamic and assigns a central role to agents’ expectations in the determination of current macroeconomic outcomes. Finally, they are stochastic since the economic framework is affected by random shocks (i.e. demand, supply and asset purchases shocks).
Every standard DSGE model must specify three elements: preferences, technology and institutional framework. Regarding the preferences, the model must specify the objectives of economic agents, for example the maximization of the utility function for households and the maximization of the profit for firms. Regarding the technology, the model must specify the productive capacity of the agents, represented for the firms by the production function plus some costs of adjustment of capital stocks. Lastly, the model must specify the institutional framework, represented by the government sector/central bank budget constraint, by the resource constraint of the economy, by fiscal and monetary policy rules. With these three elements, it is possible to solve the DSGE model in order to calculate consumption, output, inflation and other macroeconomic variables and the sensitivity of those variables in response to different shocks.

A key characteristic of DSGE models is that they share core assumptions on households and firms’ behaviors, which let them to include more elements depending on the event they need to simulate. This let researchers make their own-stylized DSGE model in order to simulate a particular event.

2.3 Basic structure of DSGE models
How Sbordone et al. (2010) assert, DSGE models used for policy analysis are formed by three main blocks: demand block, supply block and monetary policy equation. These equations derive from microfoundations, which represent the explicit assumptions about the behavior of main agents in the economy: households, firms and government. These agents interact with each other in markets that are cleared every period, leading to the general equilibrium.
Figure 2.1: Standard DSGE model

In the figure (2.1) there are represented the three main blocks of the model. The demand block represent the real activity as a function of the ex-ante real interest rate and of the expectations about the future real activity. When real interest rate is high households prefer to save money, instead to consume or invest them. Meanwhile, if the expectations about future real activity are promising then households prefer to spend their money, regardless of the real interest rate level.

As shown by the arrow from the demand block to the supply block, the real activity is a key factor to determine the level of inflation. However, the inflation is also affected by the expectations of future inflation. During economic cyclical upturns, firms have to increase wages in order to induce employees to work more, since the demand for products is higher. This increase in wages increases marginal costs, putting pressures to prices and causing inflation. Furthermore, higher inflation is expected to be in future and more it is this increase in actual prices, making inflation growing even more.

From the demand and the supply block we obtain output level and inflation level, which are used from the central bank in order to find the optimal monetary policy. The central bank sets the nominal interest rate depending on inflation and real activity. When real activity and/or inflation are growing, central bank raises the short-term interest rate. Instead, when both variables are falling central bank lowers the short-term interest rate.
Modifying the nominal interest rate, the central bank affects back real activity and so inflation, as represented by the arrow that links monetary policy block and demand block.

Thus, with these blocks it is possible to determine the three key endogenous variables: output, inflation and interest rates. In this context, it is important to consider now also the agents’ expectations about future monetary policy commitment. As Sbordone et al. (2010) assert, output and inflation tomorrow, and thus their expectations as of today, depend on monetary policy tomorrow in the same way as they do today, taking into account what will happen from then into the infinite future. The expectations influence the economy through the arrows, which flow from the monetary policy block to the demand and supply blocks, since monetary policy has a large influence on the formation of the expectations. Indeed, a key feature of DSGE models is the fact that expectations are the main channel through which monetary policy affects the economy.

Last element highlighted by the figure (2.1) is the stochastic nature. Every period, random exogenous event perturbs the equilibrium conditions in each block, making the evolution of the economy uncertain, generating in this way the economic fluctuations. These shocks are essential in order to make economy have booms and recessions. Demand shocks are caused by the willingness of households to buy more or less products from the firms. Supply shocks are caused by a change of price of products or productivity capacity of firms. Policy shocks are caused by central bank conventional monetary policies or unconventional monetary policies.

Another element that it is usually present in the standard DSGE models is the capital accumulation process, which regards the investment decisions by firms. This element would be present in the demand block of the scheme.

2.4 Elaborated DSGE models for monetary policy evaluation
As asserted before, basic DSGE models work mainly through expectations. Recalling paragraph 1.3, one of the channels through which QE generates effects in the real economy is the signaling channel, which is characterized by agents’ expectations. In fact, how Falagiarda (2013) asserts, in standard DSGE models, QE works only through signaling channel, since the representation of the financial sector is very stylized. In order to analyze the effects of QE also through other channels, such as portfolio
rebalancing channel, bank lending channel and liquidity premia channel, it is necessary to introduce specific financial, credit and liquidity frictions.

Through the introduction of financial frictions, it is possible to study the portfolio-rebalancing channel due to imperfect asset substitutability, as asserted by Tobin (1969). Furthermore, through the introduction of heterogeneous households and inefficient financial intermediation it is possible to study the bank-lending channel, as asserted by Curdia and Woodford (2008). Moreover, with the introduction of illiquid assets such as commercial paper, bank loans, etc. let DSGE models to study also the liquidity premia channel, as shown by Del Negro et al. (2011). We focus now the study on the first channel, where investors tend to rebalance their asset portfolios whenever the supply of the different types of assets changes. Indeed, QE policies vary the relative supply of assets of different maturities, inducing movement in their prices, which may influence ultimately also the aggregate demand. The central bank, through the purchase of particular asset, reduces the amount of that asset held by households, in exchange of risk-free reserves, thus creating easier conditions for economic recovery through the monetary transmission mechanism.

### 2.5 DSGE model for portfolio rebalancing channel’s evaluation

The model presented here recalls Chen et al. (2012), Harrison (2012) and Falagiarda (2013) models and has the aim to evaluate government-bond purchase programs. Common elements in these studies are the imperfect asset substitutability generated by the bond adjustment costs and a feedback channel from the term structure to the macroeconomy. Anyway Falagiarda’s (2013) model, differing from Chen et al. (2012) and Harrison (2012), introduces a secondary market for bond trading, which permits to analyze zero-coupon government bond of different maturities (short and long term). Furthermore, Falagiarda (2013) relies on a representative agent setting, avoiding the
differentiation between restricted and unrestricted agents. Finally, compared to Harrison (2012b) model, Falagiarda (2013) prefers to introduce portfolio adjustment frictions in the budget constraint than in the utility function of households.

Summarizing, the key elements of this DSGE model are:

1. Representative agent setting
2. Stylized central bank’s balance sheet
3. Endogenous term structure

Next paragraphs (2.5.1-2.5.7) will show microfoundations of the model regarding the behavior of households, firms and central bank and exogenous processes regarding certain variables. Paragraphs (2.5.8-2.5.13) will show the main equilibrium conditions of the model and particular characteristics that makes this DSGE model differ from the standard ones.

2.5.1 Households
The first step is to model households behavior, which represents the demand block of DSGE models. As asserted before, this model utilizes a representative household, whose infinity stream utility function depends on consumption $C_t$, real money balances $\frac{M_t}{P_t}$ and labor $L_t$:

$$U_t = \sum_{t=0}^{\infty} \beta^t v_t^{PR} u \left( C_t, \frac{M_t}{P_t}, L_t \right)$$

Where $\beta^t$ is the intertemporal discount factor and $v_t^{PR}$ is the demand shock.

The instantaneous utility function $u \left( C_t, \frac{M_t}{P_t}, L_t \right)$ is given by:

$$u \left( C_t, \frac{M_t}{P_t}, L_t \right) = \frac{(C_t - \gamma C_{t-1})^{1-1/\sigma}}{1 - \frac{1}{\sigma}} + \frac{1}{1 - \chi} \left( \frac{M_t}{P_t} \right)^{1-\chi} - \frac{\psi}{1 + \frac{1}{\psi}} L_t^{1+\psi}$$

Where $\gamma$ is the importance of consumption habits, $\sigma$ is the elasticity of intertemporal substitution, $\chi$ is the elasticity of money demand, $\psi$ is the Frisch elasticity of labor supply (the latter measures the substitution effect of a change in the wage rate or labor supply).
The equation (2) asserts that household like consumption $C_t$ and real money balances $(M_t/P_t)$ but dislike the number of hours they spend at work $L_t$. The utility becoming from consumption depends, on one side, from consumption today and, on other side, from past consumption through the consumption habit $\gamma$.

Each agent is also subject to an intertemporal budget constraint:

$$\frac{B_t}{P_t R_t} + \frac{B_{L,t}^H}{P_t R_{L,t}} (1 + AC_t^L) + \frac{M_t}{P_t} + I_t (1 + AC_t^I) =$$

$$= \frac{B_{t-1}}{P_t} + \frac{B_{L,t-1}^H}{P_t R_t} + \frac{M_{t-1}}{P_t} + w_t L_t + q_t K_t - C_t - T_t$$

(3)

On the right side of the equation (3) we have the sources, while on the left side we have the uses of the representative household. In the time $t$, the household has some wealth to allocate, coming from the time $t - 1$, represented by the two types of zero-coupon bonds $B_{t-1}$ (money-market bonds) and $B_{L,t-1}^H$ (long-term bonds), money holdings $\frac{M_{t-1}}{P_t}$, plus wages $w_t$ and accumulation of capital $K_t$ through the rental rate $q_t$ earned in time $t$. The agent then allocates his wealth between consumption $C_t$, taxes $T_t$, money holdings $\frac{M_t}{P_t}$, the two types of zero-coupon bonds $B_t$ and $B_{L,t}^H$, and investment $I_t$. In both sides we find $P_t$ and $R_t$, respectively representing the aggregate price level and the gross rate of return. $AC_t^L$ is the portfolio adjustment cost of bonds, while $AC_t^I$ is the adjustment cost of investment of firms.

The left-hand side of the budget constraint is characterized by bonds’ pricing at their interest rates since at time $t$ households will buy those bonds, which are risk-free from their point of view, priced at the returns $R$ and $R_L$. On the right-hand side, instead, we have the secondary market for bond trading as proposed by Ljungqvist and Sargent (2004), in which long-term bonds are priced at the money-market rate. At the time $t - 1$, an agent who buys long-term government bonds and plans to sell them next period doesn’t know the return $R_t$ he will have in the time $t$ since the long-term bonds are subject to price risk prior to maturity. However, $R_t$ can be derived through the arbitrage behavior of households. Since both short-term bonds and long-term bonds will surely reach the maturity, households will sell short-term bonds and buy long-term bonds in case the return of the latter ones is lower than $R_t$ and vice-versa. This process works
because the return of the long-term bonds bought and sold with one period gap must be the same to the return of short-term bonds that mature in the same period gap.

The capital follows the standard law of motion:

\[ K_{t+1} = I_t + (1 - \delta)K_t \]  

(4)

Where \( \delta \) represents the depreciation rate of the capital stock.

Firms face quadratic adjustment costs of investment (Kim, 2000). In this way larger investment leads to more costly adjustments:

\[ AC_t^I = \frac{\phi_K}{2} \left( \frac{I_t}{K_t} \right)^2 \]  

(5)

Financial frictions are introduced through portfolio adjustment costs of bonds, which represents impediment to the arbitrage behavior that would equalize asset returns in case households want to switch between short-term and long-term bonds. Thus, intertemporal trading between bonds of different maturities is costly to each agent. Recalling Andrés et al. (2004), Tobin (1969) and Zagaglia (2009), households face quadratic adjustment costs:

\[ AC_t^L = \left[ \frac{\phi_L}{2} \left( k_L \frac{B_t}{B_{L,t}^H} - 1 \right) \right]^2 Y_t \]  

(6)

Where \( k_L \) is the steady-state ratio of long-term bond holdings relative to short-term bond holdings \( \left( \frac{B_{H,t}^H}{B_L} \right) \). In this way agents pay a higher cost whenever they want to switch higher quantity between short-term bonds and long-term bonds.

There are three main reasons to consider those financial frictions:

1. Agents are adverse to liquidity risk, requiring then a liquidity premium that is proportional to the maturity of the bond; if agents purchase long-term bonds, they will buy also additional short-term bonds to compensate the higher liquidity risk (Andrés et al., 2004)
2. Agents have preferences over bond maturities, according to the theory of preferred habitat, as Vayanos and Vila (2009) assert, implying that any change to households’ portfolio composition is costly to them
3. There are also information and bond portfolios managing costs that have to be considered in the financial frictions structure
Assuming rational expectations about future states of economy, the objective of households is to maximize their lifetime utility under the budget constraint and the capital accumulation law. To solve the maximization we must recur to the method of Lagrange multipliers:

\[
\mathcal{L} = \mathbb{E}_t \sum_{t=0}^{\infty} \beta^t \left\{ \left[ \frac{(C_t - \gamma C_{t-1})^{1-\frac{1}{\sigma}}}{1 - \frac{1}{\sigma}} + \frac{1}{1-\chi} \left( \frac{M_t}{P_t} \right)^{1-\chi} \right] + \frac{-\psi}{1 + \frac{1}{\psi}} L_t^{1+\frac{1}{\psi}} + \frac{B_{t-1}}{P_t} + \frac{B_{t-1}^H}{P_t R_t} + \frac{M_{t-1}}{P_t} + w_t L_t + q_t K_t - C_t - T_t - \frac{B_t}{P_t R_t} + q_t K_t - C_t - T_t - \frac{B_t}{P_t R_t} + q_t K_t - C_t - T_t - \frac{B_t}{P_t R_t} + q_t K_t - C_t - T_t - \frac{B_t}{P_t R_t} + q_t K_t - C_t - T_t - \frac{B_t}{P_t R_t} + q_t K_t - C_t - T_t - \frac{B_t}{P_t R_t} + q_t K_t - C_t - T_t - \frac{B_t}{P_t R_t} + q_t K_t - C_t - T_t - \frac{B_t}{P_t R_t} + q_t K_t - C_t - T_t - \frac{B_t}{P_t R_t} \right \} \right \}
\]

Where \( \lambda_t \) and \( \mu_t \) are two Lagrange multipliers, respectively representing the marginal utility of income and the marginal value of capital. While \( \lambda_t \) indicates how much the utility function at the optimum would increase if there is a small increase in income, \( \mu_t \) indicates how much the utility function at the optimum would increase if there is a small increase in capital.

The household behavior is obtained through the maximization respect different variables:

\[
\frac{\partial \mathcal{L}}{\partial C_t} = 0 \Leftrightarrow v_t^{PR}(C_t - \gamma C_{t-1})^{-\frac{1}{\sigma}} - v_t^{PR} \beta \gamma E_t (C_{t+1} - \gamma C_t)^{-\frac{1}{\sigma}} = \lambda_t
\]

\[
\frac{\partial \mathcal{L}}{\partial L_t} = 0 \Leftrightarrow v_t^{PR} \psi L_t^{\frac{1}{\psi}} = \lambda_t w_t
\]

\[
\frac{\partial \mathcal{L}}{\partial M_t} = 0 \Leftrightarrow v_t^{PR} \left( \frac{M_t}{P_t} \right)^{-\chi} + \beta E_t \frac{\lambda_{t+1}}{\pi_{t+1}} = \lambda_t
\]

\[
\frac{\partial \mathcal{L}}{\partial B_t} = 0 \Leftrightarrow \beta E_t \frac{\lambda_{t+1}}{\pi_{t+1}} = \lambda_t \frac{k_t \phi_t Y_t (k_t b_t b_{L,t} - 1)}{R_{L,t}}
\]
\[
\frac{\partial L}{\partial B_L^t} = 0 \iff \beta E_t \frac{\lambda_{t+1}}{\pi_{t+1} R_{t+1}} = \frac{\lambda_t}{R_{t,t}} + \frac{\phi_t \lambda_t Y_t \left(k_t b_t^{L_t} - 1\right)}{2 R_{L,t}} - \frac{k_L \phi_t \lambda_t b_t \left(k_t b_t^{L_t} - 1\right)}{b_{L,t} R_{L,t}}
\]

\[
\frac{\partial L}{\partial K_t} = 0 \iff \beta (1 - \delta) E_t \mu_{t+1} = \mu_t - \lambda_t \left[q_t + \phi_t \left(\frac{l_t}{K_t}\right)^3\right]
\]

\[
\frac{\partial L}{\partial I_t} = 0 \iff \beta E_t \mu_{t+1} = \lambda_t \left[1 + \frac{3}{2} \phi_t \left(\frac{l_t}{K_t}\right)^2\right]
\]

Where \(\pi_{t+1} = \frac{p_{t+1}}{p_t}\) is the inflation, \(b_t = \frac{b_t}{p_t}\) is the real stock of money-market bonds, \(b_{L,t} = \frac{B_{L,t}}{p_t}\) is the real stock of long-term bonds. All those equations represent the first order conditions, or Euler equations, with respect to consumption (8), labor (9), money (10), money-market bonds (11), long-term bonds (12), capital (13) and investment (14).

### 2.5.2 Firms

The second step is to model the firms’ behavior, which represent the supply block of DSGE models. As asserted before, each firm is in a monopolistically competitive market, sells differentiated final good, and has the final aim to maximize its profit.

The production function of the firms is represented by a Cobb-Douglas function:

\[
Y_t = A_t K_t^\alpha L_t^{1-\alpha} - \Phi
\]

Where \(\alpha\) is the share of capital used in the production, \(\Phi\) is a fixed cost that makes profit equal to zero in the steady-state and \(A_t\) is the technology.

In order to model the firms’ behavior, we need to introduce before the adjustment costs function, the demand function and the profit function of each firm.

Recalling Rotemberg (1982), firms face quadratic price adjustment costs meaning that a higher price switch leads to more than proportional adjustment costs:

\[
AC_t^p = \frac{\phi_p}{2} \left(\frac{P_t(j)}{P_{t-1}(j)} - \pi\right)^2 Y_t
\]

Where \(\phi_p\) is the degree of nominal price rigidities. The equation (20) asserts the firm will not always choose to charge the optimal price since it is assumed to face costs to changing its price. The costs can be classified into two groups:
1. Costs of physically changing posted prices
2. Costs resulting from the negative reaction of its customers

Differing from Calvo’s model (1983), in the Rotemberg’s model there is no price dispersion, meaning that all firms face the same problem of changing prices. Thus, they will set the same price and produce the same quantity of each differentiated good. Furthermore in the Rotemberg’s model the cost of nominal rigidities consists in a wedge between aggregate demand and aggregate output; instead in the Calvo’s model the cost of nominal rigidities through price dispersion consists in a wedge between aggregate hours and aggregate output (Ascari, Rossi; 2009).

The demand function faced by each firm is:

\[ Y_t(j) = \left[ \frac{p_t(j)}{\bar{p}_t} \right]^{-\theta_t} Y_t \] (17)

Where \( \theta \) is the elasticity of substitution between different final goods. The equation (17) shows that demand of each single good is negatively related to the price of the good itself and positive related to the general output level.

The elasticity of substitution of demand \( \theta_t \) varies over time around a mean \( \theta \):

\[ \theta_t = \theta + v_t^{MU} \] (18)

Where \( v_t^{MU} \) is a mark-up shock.

The profit function of each firm is:

\[ P_t \Pi_t(j) = P_t(j)Y_t(j) - W_tL_t(j) - P_tq_tK_t(j) - P_tAC^p_t \] (19)

Given the wage \( w_t \) and the rental rate of capital \( q_t \), the problem of every firm is to choose \( L_t(j), K_t(j), P_t(j) \) in order to maximize has the final target to maximize the sum of expected profits:

\[ E(0) \sum_{t=0}^{\infty} \beta^t \frac{\lambda_{t+1}}{\lambda_t} \{ P_t(j)Y_t(j) - W_tL_t(j) - P_tq_tK_t(j) - P_tAC^p_t \} \] (20)

To solve the maximization we must recur to the method of Lagrange multipliers:

\[ \mathcal{L} = E(0) \sum_{t=0}^{\infty} \beta^t \frac{\lambda_{t+1}}{\lambda_t} \left\{ \begin{array}{c}
\left[ P_t(j)Y_t(j) - W_tL_t(j) + P_tAC^p_t \right] + \\
- P_tq_tK_t(j) - P_tAC^p_t \\
- P_tMC_t(j)[Y_t(j) - A_tL_t^{1-\alpha}K^\alpha - \Phi]
\end{array} \right\} \] (21)
Where $MC_t(j)$ is the real marginal cost, which is equal to:

$$MC_t(j) = P_t(j) \left( 1 - \frac{1}{e_t^Y} \right)$$  \hspace{1cm} (22)

Where $e_t^Y$ is the output demand elasticity.

The FOCs of the Lagrangian respect to $P_t(j), K_t(j), L_t(j)$ are:

$$\frac{\partial \mathcal{L}}{\partial P_t(j)} = 0 \iff \left[ (1 - \theta_t) + P_tMC_t(j)\theta_t \frac{1}{P_t(j)} \right] \gamma_t(j) = \frac{\partial AC_t^P}{\partial P_t(j)} P_t + \beta E_t \frac{\lambda_{t+1}}{\lambda_t} \frac{\partial AC_{t+1}^P}{\partial P_t(j)} P_{t+1}$$ \hspace{1cm} (23)

$$\frac{\partial \mathcal{L}}{\partial K_t(j)} = 0 \iff P_tq_t = \alpha \left( \frac{Y_t + \Phi}{K_t} \right) MC_t(j)$$ \hspace{1cm} (24)

$$\frac{\partial \mathcal{L}}{\partial L_t(j)} = 0 \iff W_t = (1 - \alpha) \left( \frac{Y_t + \Phi}{L_t} \right) MC_t(j)$$ \hspace{1cm} (25)

Imposing symmetry between firms (so equal marginal costs) and combining above equations, we can calculate the inflation equation:

$$\frac{1}{e_t^Y} = \frac{1}{\theta_t} \left\{ \frac{1 - \phi_P(\pi_t - \pi)\pi_t}{\lambda_t} + \frac{1}{\lambda_{t+1}}(\pi_{t+1} - \pi)^2 \frac{Y_{t+1}}{Y_t} \right\}$$ \hspace{1cm} (26)

This formula measures the gross price markup over the marginal cost. Manipulations of the equation (26) lead to the standard New Keynesian Phillips curve, which represents the supply pillar of the model.

### 2.5.3 Central bank

The third step of modeling regards the central bank role, which represents the monetary policy block of DSGE models. The elements that has to be modeled are the budget constraint, the fiscal policy and the monetary policy.

This model does not distinguish central bank and government balance sheets, considering them as a whole. The central bank budget constraint is given by:

$$\frac{B_t}{P_tR_t} + \frac{B_{Lt}}{P_tR_{Lt}} + \frac{\Delta_t}{P_t} = \frac{B_{t-1}}{P_t} + \frac{B_{Lt-1}}{P_tR_t} + G_t - T_t$$ \hspace{1cm} (27)

Where $G_t$ is government spending and $\Delta_t$ is the change in the central bank balance sheet, which represents the quantity of money introduced into the system through asset purchases:
\[ \frac{\Delta_t}{P_t} = \frac{M_t - M_{t-1}}{P_t} - \left[ \frac{B^{CB}_{L,t}}{P_t R_{L,t}} - \frac{B^{CB}_{L,t-1}}{P_t R_t} \right] \]  

(28)

Where \( B^{CB}_{L,t} \) is the central bank’s holdings of long-term government bonds, which represent a fraction of the total amount of long-term bonds:

\[ B^{CB}_{L,t} = x_t B_{L,t} \]

(29)

The other fraction \( 1 - x \) represents the long-term government bonds in households’ portfolios:

\[ B^H_{L,t} = (1 - x_t) B_{L,t} \]

(30)

The asset purchased pursued by the central bank have the aim to modify this fraction, acquiring more long-term bonds from the hands of households.

We consider then a passive fiscal policy rule, where the amount of taxes depends on the total government’s liabilities, in order to prevent the emerge of inflation as a fiscal phenomenon:

\[ T_t = \psi_0 + \psi_1 \left[ \frac{b_{t-1}}{\pi_t} - \frac{b}{\pi} \right] + \psi_2 \left[ \frac{b_{L,t-1}}{R_t \pi_t} - \frac{b_L}{R \pi} \right] \]

(31)

Where \( \psi_0 \) is the steady-state level of \( T_t \). Looking at the equation (31), the amount of taxes varies depending on the level of public debt respect to the steady-state level. Thus, taxes are not allowed to move independently to the stock of government bonds.

Regarding the monetary policy, the central bank sets the money-market rate \( R_t \) through the Taylor (1993) rule:

\[ \log \left( \frac{R_t}{R} \right) = \alpha_R \log \left( \frac{R_{t-1}}{R} \right) + \left( 1 - \alpha_R \right) \left[ \alpha_\pi \log \left( \frac{\pi_t}{\pi} \right) + \alpha_Y \log \left( \frac{Y_t}{Y} \right) \right] + \varepsilon^R_t \]

(32)

Where \( \varepsilon^R_t \) is an i.i.d. (independent identically distributed) shock with zero mean and standard deviation \( \sigma_R \); \( \alpha_R, \alpha_\pi \) and \( \alpha_Y \) represent the response of \( R_t \) with respect to lagged \( R_t \), inflation and output. Therefore, the level of money-market rate decided by the central bank depends on the deviation of inflation and output respect to the steady-state levels and on previous money-market rate.

2.5.4 Market clearing

The last element to model is the market clearing of the economy:
\[ Y_t = C_t + G_t + I_t (1 + AC_t^I) + AC_t^P + \frac{B_{t,t}^H}{R_{t,t}} (AC_t^I) \] (33)

The total output \( Y_t \) of the economy is utilized in consumption \( C_t \), government spending \( G_t \), investment \( I_t \) plus linked adjustment costs \( AC_t^I \), price adjustment costs \( AC_t^P \) and bond adjustment costs.

### 2.5.5 Exogenous processes

The exogenous variables of this model are \( x_t, G_t, b_{L_t}, A_t, v_{t,PR}, v_{t,MU} \) which follow an AR(1) logarithmic processes. The abbreviation AR(1) process stands for auto-regressive process of order one, meaning that the exogenous variable that responds to this process is regressed on its one-period lagged values. These processes permit to the shocks to have persistent effects in the time (in theory for an infinite number of periods).

\[
\log \left( \frac{x_t}{X} \right) = \phi_x \log \left( \frac{x_{t-1}}{X} \right) + \varepsilon^x_t \tag{34}
\]

\[
\log \left( \frac{G_t}{G} \right) = \phi_G \log \left( \frac{G_{t-1}}{G} \right) + \varepsilon^G_t \tag{35}
\]

\[
\log \left( \frac{b_{L,t}}{B_L} \right) = \phi_{BL} \log \left( \frac{b_{L,t-1}}{B_L} \right) + \varepsilon^{BL}_t \tag{36}
\]

\[
\log \left( \frac{A_t}{A} \right) = \phi_A \log \left( \frac{A_{t-1}}{A} \right) + \varepsilon^A_t \tag{37}
\]

\[
\log v_{t,PR} = \phi_{PR} \log v_{t-1,PR} + \varepsilon^{PR}_t \tag{38}
\]

\[
\log v_{t,MU} = \phi_{MU} \log v_{t-1,MU} + \varepsilon^{MU}_t \tag{39}
\]

Where the \( \phi_l \) variables represent the persistence of the shock in the time, \( \varepsilon^l_t \) are an i.i.d. shocks with zero mean and standard deviation \( \sigma_t \). The variables are the denominators of the logarithms are the steady-state values.

### 2.5.7 Equilibrium conditions

The three main equilibrium conditions connected to the three blocks of standard and elaborated DSGE models are:

1. New-Keynesian IS curve from the demand block
2. New-Keynesian Phillips curve from the supply block
3. New-Keynesian Taylor rule from the monetary policy block
Those curves are obtained through the log-linearization methodology (Appendix). The next paragraph shows simplified log-linearized version of New-Keynesian IS and Phillips curve, in order to explore the dynamic connection between the different macro-variables, such as output, inflation and interest rates.

2.5.8 New-Keynesian IS curve
How Sbordone et al. (2010) assert, by combining the equations (8) and (11) it is possible to establish the negative relationship between the interest rate and desired consumption that defines the demand block of the model. For simplicity, we consider $\gamma = 0$ where households have no habit in consumption and no bond adjustment costs:

\[
\lambda_t = v_t^{PR} C_t^{-1/\sigma} \tag{40}
\]

\[
\lambda_{t+1} = v_{t+1}^{PR} C_{t+1}^{-1/\sigma} \tag{41}
\]

\[
\beta E_t \frac{\lambda_{t+1}}{\pi_{t+1}} = \frac{\lambda_t}{R_t} \tag{42}
\]

Combining the equations (40, 41, 42) we obtain the Euler equation for consumption:

\[
\frac{1}{C_t^{1/\sigma}} = E_t \left[ \frac{\beta v_{t+1}^{PR}}{v_t} \frac{1}{C_{t+1}^{1/\sigma}} \frac{1}{\pi_{t+1}} \frac{R_t}{\pi_t} \right] \tag{43}
\]

The equation (43) shows that the desired consumption increases when the real interest rate decreases, when the expected future consumption increases and when households become more impatient ($v_{t+1}^{PR}$ decreases).

We can log-linearize the (43) equation around the steady state:

\[
\tilde{c}_t = E_t \tilde{c}_{t+1} - \sigma (\tilde{i}_t - E_t \tilde{\pi}_{t+1} + E_t \tilde{\nu}_{t+1}^{PR} - \tilde{\nu}_t^{PR}) \tag{44}
\]

Where $\tilde{c}$ and $\tilde{i}$ denote the percentage deviations of consumption and of nominal interest rate from their steady-state levels, $\sigma$ denote the constant intertemporal elasticity of substitution, $\tilde{\nu}$ denotes the percentage deviation of demand shock.

The (44) shows that the actual consumption is positively related to the future expected consumption and to the future expected inflation, while it is negatively related to the actual interest rates.
We consider now a simplified clearing market condition, where output is equal to consumption:

\[ Y_t = C_t \]  

Combining the log-linearized version of Euler equation for consumption and the market clearing condition, we have the New-Keynesian IS curve:

\[ y_t = E_t \tilde{y}_{t+1} - \sigma(\tilde{i}_t - E_t \tilde{\pi}_{t+1} + E_t \tilde{v}_{t+1} - \tilde{v}_t^{PR}) \]

This curve is dynamic because it contains current and future deviations from the steady state and forward looking because contains expected next period income and inflation. How Sbordone et al. (2010) assert, households decide their consumption (and so the income) today, depending on the expected future consumption and inflation. If households expect to consume more in future, they will rather consume more now. Furthermore they will consume more today if they expect a higher inflation in the next period. Households also consider actual interest rates decided by the monetary policy authority, since higher rates lead households to save money in order to use them for future consumption and vice versa.

Anyway, how Grobl and Fritsche (2010) assert, the equation (46) does not represent the equality of aggregate investment and aggregate saving. In fact a dynamic version of the IS curve would link the current IS equilibrium to its future values. Grobl and Fritsche (2010) try to calculate the real IS-curves and find out that as long the model is based on the representative agent assumption, they obtain an infinite number of different IS-curves, each one linked to a particular period which establishes a relationship between an infinite sequence of incomes and interest rates. This poses serious problems to both the analysis of the time path of incomes and interest rates as well as for policy responses. However, they also observe that each two subsequent IS-curves are related by the Euler equation. In synthesis, the best way to describe the dynamics of the model is through the first-order conditions of households optimization together with the Phillips curve.

### 2.5.9 New-Keynesian Phillips curve

The New-Keynesian Phillips curve of Rotemberg’s model is derived by log-linearizing the inflation equation (26) around the steady-state. Considering a simplified version
where the steady-state inflation is equal to zero, the New-Keynesian Phillips curve is equal to:

$$\hat{\pi}_t = \beta E_t \hat{\pi}_{t+1} + \frac{\theta_t - 1}{\phi_p} \hat{m}c_t$$  (47)

Where $\theta_t$ is the elasticity of substitution of demand (which includes also mark-up shocks) and $\phi_p$ is the degree of nominal price rigidities.

As the equation (47) asserts, the actual inflation is directly proportional to the discounted expected future inflation, to the elasticity of substitution between goods and to marginal cost deviation. The actual inflation is instead inversely proportional to the frequency of price adjustment.

How Sbordone et al. (2010) assert, the marginal cost depends on the level of aggregate activity, among other factors. In this process, higher economic activity leads to higher wages and marginal costs. This process leads the firms to increase their prices, so boosting the inflation. In synthesis, higher marginal cost deviation lead to higher anticipated inflation today.

The two main assumptions behind the (47) are the monopolistic behavior (imperfect competitions) and sticky prices. The first assumption lets firms have some power over the price setting on their products, while the second allows firms to choose prices infrequently, which are unable to adjust immediately in response to the shocks hitting the economy. This permits the central bank to have influence over the real activity. In fact if prices are sticky expected inflation will not change at the same path of nominal interest rates decided by the central bank, with the consequence that real interest rates will change as well, affecting real activity since consumption and investment will change.

Both traditional and New-Keynesian Phillips curves explain how inflation is determined. While in the traditional Phillips curve it is the unemployment rate that drives inflation, in the New-Keynesian Phillips it is the marginal cost that drives inflation. However, how Sill (2011) shows, the unemployment rate does not appear to be highly correlated with measures of marginal cost. Moreover, the simple correlation between the two series is about zero. Another difference between regards the time perspective of the curves. While the traditional Phillips curve is backward-looking, where past inflation matters for the determination of current inflation, the New-
Keynesian Phillips curve is forward-looking, where expected future inflation matters for the determination of current inflation and where past inflation is not a relevant factor in determining current inflation. Furthermore, while the traditional Phillips curve is derived by empirical studies, microfoundations derive the New-Keynesian Phillips curve.

### 2.5.10 Taylor rule

In DSGE models, the short-term interest rate is set by the central bank. Taylor (1993) asserts that central bank modifies interest rates when inflation and/or output are deviated from the baselines levels. The log-linearized version of Taylor rule is:

\[
\bar{i}_t = \alpha_R \bar{i}_{t-1} + (1 - \alpha_R) [\bar{\pi}_t^* + \alpha_\pi (\bar{\pi}_t - \bar{\pi}_t^*) + \alpha_\gamma (\bar{y}_t - \bar{y}_t^*)] + \epsilon_t^R \tag{48}
\]

Where \(\bar{\pi}_t^*\) and \(\bar{y}_t^*\) are the baselines for the inflation and output. The equation (48) says that if inflation and output rise above their baseline levels, the central bank will raise the nominal interest rate over time above its own baseline by the amounts dictated by the parameters \(\alpha_\pi\) and \(\alpha_\gamma\) and at speed that depends on \(\alpha_R\). Higher rates have a depressive effects on the economy, cutting demand, marginal costs and inflation. In this view we can consider \(\bar{\pi}_t^*\) and \(\bar{y}_t^*\) as inflation and output targets.

As Sbordone et al. (2010) assert, \(\bar{y}_t^*\) represents the level of output that would prevail in the economy if we could eliminate all distortions, i.e. force all firms to behave competitively rather than as monopolists and allow them to change their prices freely which would grant a full employment level. Anyway, when output is at its efficient level, inflation is still not stable due to mark-up shocks. It is the task of the monetary policy institution maintain the output and inflation closest to their efficient levels through the adjustment of interest rates.

While in the traditional LM curve the central bank sets the level of money supply directly, in the Taylor rule the central bank merely reacts to variations of economic conditions in normal times since it does not try to affect the economic output as much as in the traditional IS-LM scheme.

### 2.5.11 Imperfect asset substitutability

The new factor that distinguishes this DSGE model to the standard ones is the portfolio-rebalancing channel of QE. Since households face adjustment costs when they want to
switch between long-term and short-term bonds, the central bank can affect long-term interest rate through the asset purchases.

Combining the log-linearized version of the first order conditions of short-term and long-term bonds (11) and (12), we obtain:

$$\tilde{i}_{L,t} = \tilde{i}_t + \eta_1 E_t \tilde{i}_{t+1} + \eta_2 E_t \tilde{h}_{t+1} - \eta_3 E_t \tilde{\phi}_t - \eta_4 \phi_t (b_t - b_H^t)$$  \hspace{1cm} (49)

Where $\eta_t$ are convolutions of the parameters obtained through the log-linearization methodology. The equation (49) asserts that the long-term interest rate is influenced, apart by short-term rates and inflation, also by the volume of long-term bonds held by households and by the volume of short-term bonds. More precisely, how Falagiarda (2013) asserts, the long-term rate depends positively both on the volume of long-term bonds held by households and on the volume of short-term bonds because of the imperfect asset substitutability. In other words, an increase in the relative supply of long-term bonds will increase the spread between the short-term and the long-term bonds; similarly, an increase in the relative supply of short-term bonds will increase the short-term rates leading to higher long-term rates through the term structure of the financial markets. Thus, long-term bonds’ purchases by the central bank should reduce the supply of these for the households, leading to a reduction of the long-term rates. This process recalls the portfolio-rebalancing channel of QE policies.

2.5.12 Term premium structure

The interest rate spread can be calculated by taking the difference between the long-term rate and the short-term rate. How Rudebusch, Sack and Swanson (2007) assert, this spread can be furthermore divided into an expected-rate component that reflects the anticipated average future short rate for the maturity of the bond and a term-premium, which represents the extra that investors require since they are investing their money into long-term bonds instead of short-term bonds. These elements can be summarized in the following formula:

$$i_{L,t} = \frac{1}{N} \sum_{j=0}^{N-1} E_t i_{t+j} + \xi_t$$  \hspace{1cm} (50)

Where $\xi_t$ is the term premium, which is the deviation of the long-term rate from the level consistent with the expectations hypothesis. In the figure (2.2) we can see the decomposition of assets’ yields into the risk-neutrality component and into the term
premium component. More precisely the figure shows that the term premium has dropped together with the coupon yields during last years, meaning that the relevant part of long-term interest rates has been characterized by the expectation element.

**Figure 2.2: Yield and term premium**

The key factor of this model is the endogenous term structure, developed originally by Falagiarda and Marzo (2012) in a DSGE model. The endogenous structure derives from the presence of bonds with different maturities and portfolio adjustment frictions. This process generates a time-varying term premium, whose dynamics can be studied in a better way than in standard DSGE models and without recurring to higher order approximation of the simple first-order log-linearized solution.

Regarding the relationship between the term premium and macroeconomic effects, Rudebusch, Sack and Swanson (2006) assert that a decline in the term premium has typically been associated with higher future GDP growth because makes financial market conditions more accommodative for borrowers. Of course this process does not work in a DSGE model without financial frictions, like in the framework of Eggertsson
and Woodford (2003), where injecting reserves in exchange for longer-term securities is a neutral operation.

2.5.13 Feedback channel
Through the modification of the term structure, the central bank can influence real economy through the feedback channel. This channel can be obtained analytically through the combination of the log-linearized version of the first order conditions of consumption (8), short-term bonds (11), which develop the Euler equation from consumption, and long-term bonds (12). Recalling Falagiarda (2013), the feedback channel is given by:

\[ \tilde{c}_t = \eta_5 E_t \tilde{c}_{t+1} + \eta_6 E_t \tilde{\pi}_{t+1} + \cdots - \eta_7 \tilde{\tau}_t - \eta_8 \tilde{L}_{t} \]

Where \( \eta_l \) are convolutions of the parameters obtained through the log-linearization methodology. This formula asserts that the consumption and, through general equilibrium forces, all the macro variables are affected by the term structure of interest rates in the model and not only by short-term interest rates in standard DSGE models.

2.6 Model in function and impulse responses
The long-term bonds purchases by the central bank alters the holdings of assets of different maturities and so the yields, modifying the consumption through the feedback channel. Through the market clearing condition also the other variables are influenced, like government spending and aggregate investment, which in turn affect the output. Therefore, output influences inflation through the New-Keynesian Phillips curve, due to increase of marginal costs for firms and inflation expectations. Finally, output and inflation influence the Taylor rule, which determines the short-term interest rates. In this context, autoregressive processes also influence the general dynamics.

Through this DSGE model, Falagiarda (2013) evaluates long-term government bond purchases made during LSAP II by the Federal Reserve in 2010, which bought $600 billion long-term treasuries, in addition to the reinvestment of $300 billion from earlier revenues from MBS.
The figure (2.3) shows the impact of Fed’s asset purchases on main macroeconomic variables. These charts represent impulse responses calculated as percentage deviations from the steady state. Asset purchases pursued by the Fed reduce the amount of long-term bonds at the disposal of households by about 23%, leading to a reduction to long-term bonds supply, pushing down the long-term rate and the term premium structure by about 47 basis points. The decrease of both terms is similar since the short-term rate is constrained to ZLB. Finally the feedback channel increases the output by 0.69% and the inflation by 0.28%. The simulation results are realistic and consistent with other studies characterized by different simulation techniques (i.e. VAR, event studies and other DSGE models).

**Figure 2.4: Persistence of the shock variation**

Source: Falagiarda (2013)
Furthermore, if we increase the persistence of the shock as in the figure (2.4), the persistence of the response of long-term rate and term structure increase as well. Regarding output and inflation, both their persistence and magnitude increase. This event happens because price rigidities make firms to move their prices more aggressively and so increasing the impact on inflation and output. We can see also that inflation responds more aggressively than output always because of the presence of these price rigidities.

**Figure 2.5: Constrained vs. unconstrained policy rate**

![Constrained vs. unconstrained policy rate](image)

Source: Falagiarda (2013)

Another analysis is worth to do regards the comparison between the constrained vs. unconstrained short-term rate. As the figure (2.5) shows, when the short-term rate is not constrained to zero and follow a typical Taylor rule, the effects on all variables are smaller since the impact of asset purchases is mitigated by the increase of short-term rate through the Taylor rule, which in turn increases long-term rate, term premium, output and inflation.
Figure 2.6: Bond adjustment costs variation

Last analysis regards the impact of financial frictions on the macroeconomic variables. As the figure (2.6) shows, increasing bond adjustment costs it increases the degree of imperfect asset substitutability between short-term and long-term bonds. This leads to higher magnitude effect on term premium, inflation and output.

2.7 Portfolio rebalancing channel in Quantitative easing and Credit easing

The results of this simulation prove the effectiveness of Quantitative easing policies through the portfolio-rebalancing channel. More precisely, Fed asset purchases during LSAP II helped to reduce long-term rates and boost inflation and aggregate output for the US economy thanks to an aggressive and long persistence policy. However, it is fair to say that LSAP I had stronger portfolio-rebalancing channel than LSAP II due to stronger imperfect asset substitutability since the assets purchased by the Fed were not long-term government bonds but MBS and GSE, characterized by higher credit and liquidity risks. Through these purchases, Fed has been able to affect both the term premium and the credit-risk premium, thus making a higher impact on inflation and output. However, to evaluate more correctly a credit easing policy we must also model credit frictions in financial intermediaries sector and households heterogeneity (i.e. constrained vs. unconstrained or lenders vs. borrowers households).

Thus, the portfolio-rebalancing channel is not the only channel that has to be considered when evaluating unconventional monetary policies as a whole. Indeed Bauer and
Rudebusch (2011) estimate that the portfolio rebalance channel accounted for half of the LSAP’s effects. In fact regarding the Japan’s QE 2001-06 experience, Ugai (2007) asserts that quantitative easing helped reducing yields through the portfolio-rebalancing channel but its effects on economic activity and inflation were smaller than Fed’s ones, most probably because of the dysfunctional banking sector and banks’ deleveraging, which has impaired the functioning of the bank lending channel.

Another important result of this simulation regards the unconstrained policy rate results, which prove that quantitative easing policy is more effective at the zero-lower bound. This reinforces Bini Smaghi (2009) and Borio and Disyatat (2009) statements about the fact that the quantitative easing policy has to be conducted at the zero-lower bound only since it forces banks to make more loans instead of depositing reserves at the central bank.

2.8 Other transmission channels captured and limits of DSGEs
In the economic literature, there are several studies on QE and credit easing policies made through the DSGE models. As asserted in the paragraph (2.2), the flexibility of DSGE models let the researchers to study different transmission channels such as the portfolio-rebalancing channel, the bank-lending channel and the liquidity premia channel.

Gertler and Kiyotaki (2010) elaborated a DSGE model with credit intermediation and credit frictions, in order to illustrate how credit market disruptions influence real activity and to show how the central bank intervention in these markets can mitigate the crisis. In their model, it is possible to distinguish two types of credit frictions. Credit frictions generated by the agency problem between borrowers and lenders, which makes the cost of external finance higher than the internal finance one. Furthermore, the size of this premium depends on the borrower balance sheet’s condition. During periods of crisis, the balance sheet condition of the borrower deteriorates, thus leading to a higher external finance premium. Therefore, borrowers face a higher cost of credit, leading to an increase in credit spreads between long-term government bonds and rated asset classes. In last instance, this process deteriorates even more the borrower’s financial situation and depressing the real activity. Another credit friction is introduced by the idiosyncratic liquidity shock, which let banks to have surplus and deficits of funds. This element is necessary to model the inter-bank market, where banks in surplus of funds
lend to banks in deficit of funds. During periods of crisis, banks have problem to transfer funds between each other, thus creating disruptions in the inter-bank market. Through this model, Gertler and Kiyotaki (2010) try to capture the bank-lending channel of balance sheet policies. They find out that the Fed intervention slows the decline of output by almost one third.

Del Negro et al. (2011) implemented the liquidity premia channel in their DSGE model. They hypothesize that firms can sell, every period, only up to a certain fraction of the illiquid assets, such as equity holdings, commercial paper, bank loans, etc. in their balance sheets. The central bank can change the composition of liquid and illiquid assets in the balance sheets through balance sheet policies. They find out that the increased liquidity in the markets provided by Fed monetary policy had a very high impact, with output and inflation dropping by 50% more. Reason of this relies on the nominal rigidities (which is base hypothesis for most DSGE models) and zero-lower bound. The fact that the central bank cannot lower more interest rates, together with price rigidities, cause investment, consumption and output to drop. Through the unconventional policies, the central bank can increase the liquidity and thus consumption and output.

As it is possible to see, every DSGE model attempts to analyze output and inflation dynamics through base drivers that characterize basics DSGE models, plus structures regarding the financial markets, banking system, liquidity constraints and so on. Thus, researchers need to elaborate different types of DSGE if they want to analyze the plurality of economical drivers that conduct the economy.

However, DSGE models are still object of research and development by central banks across the world. Recalling Tovar (2009), the modelling aspects and computing tools to solve them are completely new. Therefore, since the methods for solving and estimating are different, policy makers limited their acceptance to discuss their policies and to forecast economic variables. Some economists, like Sims (2006), argued that DSGE models simplify too much the reality and thus they cannot truly describe the dynamics of the data. For instance, economists still did not implement in those models the different types of financial markets present in the economy all together. Lastly, how Kocherlakota (2007) asserts, to make those models generate truly results, economists have to calibrate and not to estimate them with econometric techniques. Even with the
latter ones, researchers have to de-trend the data, eliminate the outliers, select stable periods or eliminate structural breaks.

Anyway, those models have grown over time and many central banks around the world have used them to analyze monetary policy issues, but still more work needs to be done in three areas, as Tovar (2009) points out. First, DSGE models still need to incorporate successfully relevant transmission mechanisms and sectors of the economy. Second, there is still discussion about how take those models to the data. Third, researchers have to communicate effectively the features and the implications of those models to policy makers and to the public. Those models will face an important trade-off in the future. On one side, in order to grant more realism to DSGEs, researchers have to make them more complex. On the other side, this will make those models more difficult to understand and thus to analyze and communicate. For Tovar (2009), those models cannot still perform better than traditional models like econometric ones. Therefore, for now DSGEs will be used by central banks only as complementary tools for the standard ones, while researchers will try to elaborate more complex models, which will be also easy to analyze and communicate.

**Conclusions**

The research of new theoretical models to explain the complexity of economic processes has been a challenge for many economists since years. Since the birth of modern macroeconomics with Keynes, economists have continuously elaborated new models to seek the drivers of relevant events such the Great Depression and the Oil Crisis of 1970s. Today economists and policy makers across the world are focusing their attention on the recent financial crisis and on unconventional monetary policies utilized to stop it. In this case, they keep developing new models to understand the mechanism behind this crisis and behind the relative policies undertaken. In this context, DSGE models have shown a great capability to isolate and study the different monetary policies transmission channels and to see their effects on the real economy. In our case, the DSGE model has succeeded to isolate the portfolio-rebalancing channel and has shown its effectiveness to influence the real economy. However, those models still need to be perfected in order to be used on a daily-basis by central banks as a forecasting instrument for their policies.
Bibliography


Ascari, G. and Rossi, L. (2009): “Trend Inflation and Firms Price-Setting: Rotemberg vs. Calvo”, Quaderni di Dipartimento 100, University of Pavia, Department of Economics and Quantitative Methods

Bank of Japan Annual Review of 2006


Chung, H., Laforte, J-P., Reifschneider, D., Williams, J. C., (2011): “Have we underestimated the likelihood and severity of zero lower bound events?”, Federal Reserve Bank of San Francisco, pp. 1-56


Fawley, B. W., Neely, C. J. (2013): “Four Stories of Quantitative Easing”, Federal Reserve Bank of St. Louis

Federal Reserve Annual Report of 2010


Woodford, M., (2012): “Methods of policy accommodation at the interest-rate lower bound”, Columbia university, pp.1-88
Zagaglia, P. (2013): “Forecasting long-term interest rates with a general-equilibrium model of the euro area” what role for liquidity services of bonds?”, Asia-Pacific Financial Markets

Appendix

Log-linearization methodology
The non-linearity and infinite dimensionality of the model makes the solving problem very difficult for mathematicians and computer scientists. Mainly because of these computational difficulties, macroeconomists at central banks prefer to use the log-linearization methodology around a constant steady-state. As pointed out by Tovar (2009), due to the computational burden associated with the evaluation for the solution of non-linear expectation equations of DSGE models, the empirical literature has concentrated its attention on the estimation of first-order linearized DSGE models. Though this methodology has several important drawbacks, such as the impossibility to model and study systemic risk, welfare effects, asset pricing and term and risk premiums.

Linearization consists into the conversion of non-linear difference equations into linear difference equation around a steady-state. Log stands for logarithm, that is the mathematical instrument used in order convert the original variables into percentage deviations around the steady-state.

In order to understand the log-linearization methodology, it’s important to recall the Taylor’s theorem, through which an univariate function \( f(x) \) can be expressed as a power series of derivatives about a particular point \( x^* \), where \( x^* \) belongs to the set of possible \( x \) values:

\[
f(x) = f(x^*) + \frac{f'(x^*)}{1!}(x - x^*) + \frac{f''(x^*)}{2!}(x - x^*)^2 + \frac{f'''(x^*)}{3!}(x - x^*)^3 + \cdots
\]

The same is valid for multivariate functions. In fact a multivariate function \( f(x,y) \) can be expressed as a power series of partial derivatives around a particular point \( (x^*, y^*) \):

\[
f(x, y) = f(x^*, y^*) + f_x(x^*, y^*)(x - x^*) + f_y(x^*, y^*)(y - y^*) + \cdots
\]

Where \( f_x \) and \( f_y \) are partial derivatives of the function with respect to \( x \) and \( y \).
Through the Taylor’s theorem it’s possible to study function near its steady-state without caring about all the function structure. This is simplifies a lot the study of some particular functions that its solution are hard to find.

The main steps for log-linearizing a function are three:

1. Calculate the logarithms of the equations
2. Calculate the first order Taylor series expansion about the steady-state
3. Simplify so that everything is expressed in percentage deviations from steady-state

Considering for example a Cobb-Douglas production function:

\[ y_t = a_t k_t^\alpha n_t^{1-\alpha} \]

Can be log-linearized to obtain the following function:

\[ \tilde{y}_t = \tilde{a}_t + \alpha \tilde{k}_t + (1 - \alpha) \tilde{n}_t \]

Where the tilde variables \( \tilde{x} \) are the percentage deviation of \( x \) around the steady-state \( x^* \):

\[ \tilde{x} = \frac{x - x^*}{x} \]

This log-linearization instrument can be used to log-linearize all equilibrium equations of the model in order to determine, in final goal, output, inflation and long-term interest rates.