



Department  
of Economics and Finance

Chair of “*Corporate Finance*”

***An application of the Net Present Value: a case study on the fiscal  
implications of an Italian newborn***

*Prof. Ugo Zannini*  
Supervisor

*ID number 222891*  
*Valeria Maiellare*  
Candidate

Academic year 2019/2020

# INDEX

<b>INTRODUCTION</b> .....	2
<b>CHAPTER I:</b>	
<b>AN HISTORICAL PERSPECTIVE OF THE NET PRESENT VALUE UNTIL TODAY</b>	
1.1. The concept of investment decisions and the ban on the interests.....	4
1.2 Early contributions to NPV as a method for the valuation of the annuities.....	7
1.3 The crucial contribution of Gottfried Wilhelm Leibniz to the development of NPV and his successors...9	
1.3 The evolution of the contemporary use of NPV.....	10
1.5 The general rules for applying the Net Present Value rule today.....	14
<b>CHAPTER II:</b>	
<b>A NET PRESENT VALUE CALCULATION: A CASE STUDY ON THE FISCAL IMPLICATIONS OF A NEWBORN IN ITALY</b>	
1.1 Introduction of the case study.....	17
1.2 Presentation of the model.....	18
1.3 Data and calculation.....	20
1.4 Results and discussion.....	23
<b>CONCLUSION</b> .....	28
<b>REFERENCES</b> .....	29

## INTRODUCTION

Corporations make financial decisions through investing in real assets, which generate income. Companies invest in lots of things – be these tangible assets, like factories, machineries and offices or intangible assets, such as patents or trademarks – as well as individuals make investments, for instance in a college education. Both companies and individuals pay for their investments by raising money and, in the process, assuming the liability to repay that money with interest later.

All these financial decisions require comparison of cash payments at different dates in order to understand how much you will have to repay to the lender if you borrow money from him or how to choose the best investment alternative to increase the value of the corporation and its current stock price years later. Using correct positive theories, which consists, as Jensen and Smith<sup>1</sup> have reported in their review of the development of the modern theory of corporate finance, in evaluating how alternative actions affect the desired outcome, is fundamental in order to take decisions that have expected and desirable outcomes.

Therefore, knowing that a dollar today is worth more than a dollar tomorrow, it is clearly important to know how much an individual need to invest today to produce a certain amount of money at the end of some years later. This is simply known as the present value or Net Present Value (NPV), if we subtract the required initial investment from the result. For instance, a manager wants to know how a decision will affect net cash flows, their riskiness and therefore the firm value in order to take the better choice among several and different alternatives.

The concept of net present value is extremely useful because it enables managers as well as investors to figure out today's value of an asset at a given simple interest rate by just adding up the net present value of all future cash flows received. The interesting point is that we can apply the concept of NPV to a reality closer to us. Everyday people, not only financial manager or investors like engineers or banks, are required to make a choice about building or not a bridge, enter in a project or following a master program. The question is always the same: will your choice bring surplus to your life or your firm in order to justify your current expenditure? The simplicity and efficacy of NPV can be exploit in different backgrounds besides corporate finance. The net present value rule states that a person should invest in projects or engage in transactions only if they have a net present value larger than zero. This key concept suggests that actual problems more attached and closer to real life, other than those cited above, can be explained through this ancient methodology. For instance, a fiscal analysis about the cash inflow and outflow of a middle-age employee in Italy or the discrepancy between the gain and expenditure from the point of view of

---

<sup>1</sup> Jensen, Smith, 1984. *The theory of Corporate Finance: A historical Overview*, pp 1-3

two economically different countries. Another prominent question and highly discussed topic can be the increasing inflow of immigrants to Western countries. Given the multitudes of conflicting opinions, it could be interesting to understand the fiscal cost or gain received from a new immigrant today in a certain country using the net present value calculation and a simple interest rate.

Compared to other management techniques, the net present value was developed and popularized comparatively late. The first chapter will present the history of the NPV from its origin until today, showing the main leading figures responsible to its expansion and discussing the reasons for its development delay. In particular the ban on interest, especially on compound interest, in religion and in philosophy is identified as a key obstacle to its development, since compound interest is crucial for calculating NPV. Only starting from 1800 the idea of valuing projects using the NPV became relevant.

Today, net present value is a globally accepted methodology for investment appraisal in companies, but its background is full of drawbacks.

The second chapter presents a case study in which I would like to investigate the fiscal implications derived from a new-born in Italy today using the net present value calculation. The analysis aims to find whether the birth of a native, being it an intangible asset and not a proper and usual investment, will represent a future gain for a nation's welfare. In other words, if your future contributions, in terms of money, will be sufficient to recover the expenditures that the state has to make during a certain time lapse. Moreover, the present value concept allows us to compare the value of two alternatives with very different timing of their cash flows. Referring to the study under consideration, the latter property is just giving us the possibility to compare the gain, or simply NPV, obtained from one person in two different moment of their life, for instance when it is young and old, and see if they are both positive or which is higher. Once again, we can definitely see how to apply the net present value beyond corporate finance.

The last section shows and discuss the results obtained from the case study with the help of a graph. A sensitiveness analysis has been also outlined.

## ***CHAPTER I***

# **AN HISTORICAL PERSPECTIVE OF THE NET PRESENT VALUE UNTIL TODAY**

### **1.1 *The concept of investment decisions and the ban on interests***

Investment appraisal - be it the proposed purchase of a new equipment or the valuation of a potential acquisition target by a company – and the sale of assets, are not modern concepts. As long as human beings have traded with each other, they have had to make reasonable investment decisions in order to be better off. The problem on how to evaluate an investment and its relative decision has therefore always existed. Nevertheless, while other concepts and methodologies of modern management can be dated back to ancient times, modern investment appraisal was adopted later in history. The latter is based on two fundamental principles:

- 1) Investment appraisal are evaluated on the basis of the net present value rule (NPV). It states that a project is measured according to its value rather than its costs and companies should accept all projects with a positive NPVs and, in case of a limited budget, companies should choose the projects with the highest NPV. This is true on the evident logic that a dollar today is worth more than a dollar tomorrow.
- 2) Since the calculation of the NPV must be based on forecasted cash flows, risk has to be considered because the riskier the project (e.g. measured by the spread of different possible outcomes) the higher the opportunity cost of capital must be.

These two principles, expressed by Prof. Dr. Stefan Behringer<sup>2</sup>, are also accompanied by another main principle used in valuing companies or parts of companies: the discounting of cash flows where the numerator represents cash flows or another financial performance indicator, whereas the denominator represents the time value of money and has the economic function of showing the best alternative to the project being evaluated. From this it follows that in NPV the interest rate is used as the opportunity cost of the capital.

---

<sup>2</sup> Behringer (2015), *op. cit.*, pp. 74-75

Behringer explain that one reason for not using the NPV during the ancient time was the ban on interest<sup>3</sup> (usury), which find its roots in antique philosophy as well as in most religious world. Originally, usury meant the charging of any kind of interest.

Although the historic discussion about usury is linked to the transfer of loans and not to the evaluation of projects, there is a close connection between the two issues because of the meaning of the opportunity cost of capital. The opportunity cost of capital depends on the investment opportunities available to investors in the financial markets as explained by corporate finance. Whenever a corporation invests cash in a new project, its shareholders lose the opportunity to invest the cash on their own. Thus, the ban on interest on loans has a direct impact on using interest rates for other purposes such as investment appraisal.

Since the ban on interest was mainly rooted in ancient philosophy as well as in most world religious, it would be remarkable to underline some of the main views on the ban on interest which are responsible for the delay of the development of NPV. One among them is that belonging to Aristotle<sup>4</sup> and reported by the Professor. Although the Greek philosopher (384-322 BC) was in favour of private property, he was against trading if it had the aim of profit making for two reasons: firstly profit distracts human beings from a virtuous and fair life and secondly profiting from the exchange of goods can only occur by harming one of the partners. Moreover, he argued that a profit should not occur if people only exchange essentials. In his opinion, money was accepted as an instrument for pure exchange not a commodity and if the desire of essential commodities has a natural limit, the wish to have money is infinite and for this reason money represents a problematic commodity. It is important to mention that in ancient Greece lending money and charging interest was not uncommon. Therefore, Aristotle's view was based only on ethical reasonings, not empirical, which lead to the delay of NPV's development.

Also, the Old Testament of the Christian Bibles in Judaism<sup>5</sup> mentions the ban on interest rates in several chapters; interest is banned for all money lending and the ban is mainly on charging interest to the poor and disadvantaged. More in general charging interest was criticised on moral grounds even if some passages in the Testament allow charging interest to non-Jews. The Talmud, the central text of Rabbinic Judaism and the primary source of Jewish religious law, is also very critical on charging interest but it allows Jewish merchants to charge interest if the terms and conditions of the interest are identical to those of other merchants charging interest. This negative connotation of interest in Judaism might also derive from *neshekh*, the Hebrew word literally meaning "a bite": it is believed to show the debtor's reaction to interest.

---

<sup>3</sup> Behringer (2015), *op. cit.*, pp. 75-77

<sup>4</sup> Behringer (2015), *op. cit.*, pp. 75-76

<sup>5</sup> Behringer (2015), *op. cit.*, p. 76

In some Christian societies and even today in many Islamic cultures, charging any interest at all was considered usury. Religious prohibitions on usury are predicated upon the belief that charging interest on a loan is a sin. The origin of the ban on interest in Christianity lies in the Judaic tradition, which Aristotle's view reinforced. The official ban on interest was codified in the First Council of Nicaea in 325 AD<sup>6</sup>. The ban referred initially to the clergy and, along the time and in particular during the time of Charlemagne, extended to all people.

Although the Church still does not allow the charging of interest, the lender's potential loss due to delayed or denied repayment was now accepted as a reason for charging interest.

Even if the usurer was persecuted by the Catholic Church only in flagrant cases, this not discharge the usurer from facing two real risks. First, the debtor could take this loan case to the courts and, by doing so, attain a non-repayment of his loan. At the end of the 12<sup>th</sup> century, the discovery of silver mines in Germany as well as the Christian expansion into Islamic Spain, cause a speculative credit boom which fed the dramatic increase of public denunciation about usury. The second risk is the eternal damnation after the Last Judgement a usurer was condemned. Since usury was tough as a mortal sin, this risk was perceived to be a real threat in the Middle Ages.

Only later, in the early 19<sup>th</sup> century, as explained by Behringer, the Catholic Church slowly renounced on its ban on interest. Moreover in 1913, the *Catholic Encyclopedia* recognized that the Catholic Church charged interest on its own property.

Before the 19<sup>th</sup> century scholastic practice only allowed charging an interest rate if the loan was given in an emergency, if the creditor could not realize another profit because of extending the loan or if the repayment of the loan was overdue.

On the contrary, at the end of the 16<sup>th</sup> century, scholars begin, despite the strength of the moral condemnation, to engage in working with interest rates for purposes like project valuation.

In the Muslim world, the charging of interest is prohibited until today for the Islamic financial system because there is the idea that one cannot profit without taking any risks.

Historically, compound interest<sup>7</sup> was considered to be the worst kind of usury and was condemned in various medieval legal systems, but compound interest is necessary in order to include the time value of money in the calculation of NPV. Only through using compound interest the NPV calculation can incorporate the following effect: one dollar today can be invested and earn money immediately.

---

<sup>6</sup> Behringer (2015), *op. cit.*, pp. 76

<sup>7</sup> Behringer (2015), *op. cit.*, pp. 77

Compound interest was already charged in Mesopotamian times, when temples offered investment and credits. On the other hand, in order to reduce the burden of debtors, the Babylonian and Sumerian authorities proclaimed debt cancellations on a regular basis.

The mathematics of compound interest were elaborated during the 16<sup>th</sup> and 17<sup>th</sup> centuries and used in business life. In 1566, Jean Trenchant published his book “*L’Arithmetique*”<sup>8</sup> in Lyons which includes a chapter about compound interest. In 1613, Richard Witt was the first who demonstrate the concept of compound interest by using the formula<sup>9</sup>  $(1+i)^n$ , including tables showing an interest rate of 10%, which was the most widely used rate at that time. Even though theoretical research for developing and refining rules, had not yet been undertaken in 17<sup>th</sup> century in England, compound interest was already common and established in merchants’ daily business transactions and life.

## 1.2 *Early contributions to NPV and a method for valuation of the annuities*

The development of the NPV was really slow and early contributions to this methodology were occurred during Romans time. While Greek philosophers refused interest, the Romans used it in their transactions. A problem that was already occurring in Roman times was the valuation of life annuities, which is similar to discounting future cash flows in an investment or valuation system. The Roman lawyer Ulpian (170-228 AD) developed a simple approach to value payments that was linked to both the age and life expectancy of the beneficiary. These calculations resulted to be useful to value the life annuity for tax purposes and the legacy of the inheritance. An approach similar to contemporary mortality tables was used. Depending on the current age of the beneficiary we have the following model<sup>10</sup> reported by the Professor Behringer:

- If the beneficiary was 20 years or younger, the current value was equivalent to 30 annual payments.
- If the beneficiary was aged between 20 and 25 years old, the current value was equivalent to 28 annual payments.
- ...
- If the beneficiary was older than 60 years, the current value was equivalent to 5 annual payments.

---

<sup>8</sup> Behringer (2015), *op. cit.*, p. 77

<sup>9</sup> Behringer (2015), *op. cit.*, p. 78

<sup>10</sup> Behringer (2015), *op. cit.*, p. 78



Although this approach is similar to the modern one, it lacks one important instrument: an interest rate. The lack of an interest rate charged would lead payments at different points in time to be comparable. In conclusion, Ulpian's model does not consider the time value of money.

More recently, once the problem of valuing life annuities has long been discussed, an important discovery, paving the way toward modern calculation, came in the form of the book "*Value of Life Annuities in Proportion to Redeemable Annuities*"<sup>11</sup> by Johan de Wit in 1671. He used an almost contemporary mortality table, stating that out of 768 nominees:

- Six will die every 6 months for the first 50 years
- Four will die every 6 months for the next 10 years
- Three will die every 6 months for the next 10 years and
- Two will die every six months for the next 7 years

For every nominee, he calculated the present value of his annuity assuming an interest rate of 4%. The valuation of life annuities, using compound interest by valuing a stream of payments, was elaborated further by Abraham de Moivre (1657-1754) and Edmund Halley (1656-1742). At this point Halley in 1761 developed a formula for the net present value of an annual payment that starts in year 1 and ends in year T with a final payment equal to<sup>12</sup>

$$\frac{x}{r-1} \cdot \left(1 - \frac{1}{r^T}\right)$$

This formula can be found in his paper "*Of compound interest*" where x is the amount of the annual payment and r stands for 1 plus the interest rate for the annuity.

Only in the 13<sup>th</sup> century the aggressive fight against the ban on interest has started, because especially Italian banks expanded their business with loans and rents. Another prominent example is the "*Liber Abaci*"<sup>13</sup> by Leonardo of Pisa called Fibonacci published in 1202. The latter used the time value of money and the net present value method for valuing cash flows. The annual payments result in a lower net present value than the quarterly installments: a soldier receives a quarterly payment of 75 or a yearly payment of 300. The difference results from the fact that the soldier can earn every month a certain amount of interest. Therefore, the more compounding periods throughout this one year, the higher the future value of the investment, so naturally, two

---

<sup>11</sup> Behringer (2015), *op. cit.*, p. 79

<sup>12</sup> Behringer (2015), *op. cit.*, p. 79

<sup>13</sup> Behringer (2015), *op. cit.*, p. 79

compounding periods per year are better than one, and four compounding periods per year are better than two. Subsequently, in 1582, the Dutch scientist Simon of Bruges'<sup>14</sup> contribution was considered to be the first written document of NPV. He published the interest tables and mentioned the formula for discounting cash flows in an appendix as a general rule for finding the most beneficial loan at a given interest.

### 1.3 *The crucial contribution of Gottfried Wilhelm Leibniz to the development of NPV and his successors*

Leibniz was born in Leipzig, Germany, in 1646 and he became the last man who was considered to possess the complete knowledge of his time. His essay "*Meditatio Juridico Mathematica de Interusurio simplice*"<sup>15</sup> ("*Legal and mathematical discussions about simple interest*", in English), cited by Behringer, several versions of which were written between 1680 and 1683, was highly relevant to the development and implementation of NPV. Leibniz began his essay writing down three fundamental principles upon which everyone should agree in a situation where one person is the borrower and the other is the lender:

- 1) Everyone who pays back an amount of money earlier than contractually necessary can claim an adequate amount of interest for it.
- 2) Payments are not necessary. Offsetting receivables and payables are also allowed.
- 3) Both parties can always agree on earlier payments and/or offsetting

Assume that the debtor repays one dollar today even though it is due in one year. Principle 1 implicitly suggest that, in the following case, the creditor has to pay interest on this amount. Since offsetting is allowed, following principle 2, the debtor can reduce the interest on his or her payment. If we assume an interest rate of 5%, the debtor has to pay one dollar and five cents of interest. This interest is, however, due one year after the original payment period. In this way, the debtor has to pay interest on the interest he or she receives from the creditor following a numerical series as long as necessary. The logic Leibniz applies is nothing other than the NPV rule.

Even though these are the conclusions of the last version of his essay, in the first version he stated that compound interest was unacceptable, showing how deep the resistance to compound interest was rooted in the general thinking of his time. Not by chance he used the term 'simplice' with interest in the heading of his paper.

---

<sup>14</sup> Behringer (2015), *op. cit.*, p. 78

<sup>15</sup> Behringer (2015), *op. cit.*, pp. 79

In the second version, he mentioned a formula equivalent to NPV that is still used today:

$$a \cdot \left( \frac{v}{v+1} \right)^z$$

Where  $a$  is the amount of debt,  $z$  is the number of years for which the money is lent, and  $1/v$  is the interest rate. If the fraction is reduced by  $v$  and  $1/v$  is written in modern notation with  $i$ , the modern NPV can be identified using the following formula:

$$a \cdot \frac{v}{(1+i)^z}$$

The calculation with compound interest was not explicitly stipulated for loans but for all any kind of other series of payments, such as annuities and investments.

Although Leibniz's work is not the invention of NPV, it logically deduces it following his three basic and easy principles. He drew the conclusion with mathematical and logical deductions; it contradicted the prevailing opinion and legal situation of the time. His approach was very modern by using compound interest because he implies that the money, which is repaid, is invested by the creditor at the interest rate, which is in turn paid by the debtor.

Nevertheless, its implementation was slow.

The Kingdom of Saxony was the first state to implement NPV by adopting Leibniz's method for public disputes in 1724, when debts were repaid ahead of time. Then, in 1804, some countries of Prussia followed while most of German states continued to use the linear calculation without compound interest until 1829. The vast majority of mathematical textbooks in Germany incorporate the Leibniz's calculation. Von Clausberg, who was regarded as the best mathematician of his time, promoted the Leibniz formula. His book became one of the most influential sources of calculation methods and was published in various editions after his death in 1751. Leibniz's work was also adopted in some parts of juridical literature. Nevertheless, other scholars were still against using Leibniz's rule, underling the deep resistance against using compound interest and thus NPV rule.

#### **1.4 *The evolution of the contemporary use of NPV***

These early and discussed contributions to the NPV methodology and the acceptance of the charge of interest did not lead to a widespread use of the NPV and its applications in finance until the beginning of the 19<sup>th</sup> century. Only in 1800 the idea of valuing projects with NPV became relevant, as more and more authors advocated its use for the appraisal of investment. At the same time the

religious prohibitions related to the charging of interest for loans had been repealed and the practice of merchandise instruments that used interest rates spread out.

It would be useful to investigate three different determinants of the time and their relative actors to understand NPV's evolution: engineering economics, political economy and academics as well consultants who popularized the method after the World War II.

Proceeding in chronological order one of the first writers to employ present value computations to non-financial investments in 1887 was an American civil engineer, A.M. Wellington. He published the second edition of his standard work on the location of railways in the US "*The Economic Theory of the Location of Railways*"<sup>16</sup> as reported by Jones and Smith. Based on his practical experience, he concluded that the decision to build a railway line needed careful reasoning because of the massive capital expenditure required. Wellington published a table showing the present value of a unit of money being invested at a specific rate of return for a given numbers of years. In the second edition of his book he extended his coverage of NPV techniques. Nevertheless, its prominent work contains two flaws: firstly, he used the loan rate of the company as a discount rate, neglecting the cost of equity; secondly, he ignored an adjustment for risk in the discount rate.

Another source of NPV in engineering economics is the textbook "*Principles of Engineering Economy*"<sup>17</sup> by Stanford professor E. L. Grant, which was established in 1930. He mentioned that interest has to be considered in order to make an appropriate appraisal in particular when payments and revenues occur on different dates. The Professor also discussed about applications of the present worth, the rate of return, and the equivalent annual cost methods for making capital budgeting decisions.

The engineering literature from the end of the nineteenth century until the end of World War II contained only scattered attempts to discuss present value or capital budgeting techniques. Engineers were more focused on improving the concepts and techniques of cost accounting. None of these attempts appears to have made a significant contribution until management thinkers popularized NPV. Nevertheless, according to Weaver, it was oil and chemical companies with a strong background in the engineering profession pioneered NPV as a management technique in the 1950s.

In 1907 there was the first reference to NPV in American economic literature by Irving Fisher in the "*The Rate of Interest*", which was revised and reissued twenty-three years later as "*The Theory*

---

<sup>16</sup> Jones, Smith (1982). An historical perspective of net present value and equivalent annual cost, pp. 104-105

<sup>17</sup> Jones, Smith (1982). op. cit., p. 105

<sup>18</sup> Behringer (2015), op. cit., p. 82

of Interest”<sup>19</sup>. Fisher presented four principles to evaluate alternative investment proposals described by Jones and Smith in their essay. These were:

- 1) The principle of maximum present value: i.e., selection based on the maximum present value determined by using the market rate of interest.
- 2) The principle of return over cost: selection of alternative whose rate of return over cost or rate of return on sacrifice exceeds the market rate of interest. The rate of return over cost concept by Fisher, as reported by Armen A. Alchian<sup>20</sup>, is developed in order to rank investment alternatives by the universally correct criterion of maximum present value. It can be defined only by reference to at least two alternative investment options. Letting subscripts denote investment options,

$$\int_0^t ([R_1[t] - E_1[t] - [R_2(t) - E_2(t)])e^{-rt} dt$$

is the difference in present worth’s of the two investment options when each is discounted at the rate  $r$ .  $R[t]$  represents the inflow stream and  $E[t]$  represents the outflow stream, both as function of time, and  $e^{-rt}$  is the discount factor to  $t$ .<sup>21</sup> The rate  $r$  which sets the difference equal to zero is Fisher’s marginal rate of return over cost. The meaning of the word “opportunity”, according to Fisher, is a choice between two alternative investments/solutions. Thus, this opportunity of option 1 over 2 at any moment  $t$ , is  $(R_1 - E_1) - (R_2 - E_2)$ , or, following the terms written before, is  $(R_1 - R_2) - (E_1 - E_2)$ . This latter expression is called by Fisher the surplus of advantages over disadvantages. The rate of discount which equates the present worth of the advantages,  $(R_1 - R_2)$ , to the present worth of the disadvantages of investment options,  $(E_1 - E_2)$  is the rate of return over cost. Fisher’s rate of return over cost always involved a comparison of two options, not only a simple discounting of one option.

- 3) The principle of comparative advantage: selection of the alternative whose returns outweigh its costs stated in present value using the market rate of interest as the discount rate.
- 4) Select among different options the one that gives a rate of return over cost equal to the rate of interest.

Probably, Fisher was the first scholar to effectively popularized NPV as a rule for investment decisions. Fisher’s contribution is a clear system for analyzing the benefit of investments. He also

---

<sup>19</sup> Jones, Smith (1982). op. cit., p. 105

<sup>20</sup> Alchian (1995). *The Rate of Interest, Fisher’s rate of Return over Costs and Keynes’ Internal Rate of return*, pp. 938-939

<sup>21</sup> Jones, Smith (1982). op. cit., p. 106

derived the separation theorem which asserts that it is sufficient that a company maximizes net present value regardless of preferences of the company's shareholders.

In 1920 Alfred Marshall discussed the concept of discounting cash flows in his book "*Principles of Economics*".<sup>22</sup> He stressed that the revenue from an investment must exceed the original cost by an amount that considers the weighting time at compound interest. Moreover, Marshall mentions that the investment must earn a surcharge for risks and potential delays. He also states that changes in the purchasing power of money must be considered. Investment are carried out in the belief that the investment can compensate the investor for his outlay.

Successively the Austrian economist Bom-Bawerk<sup>23</sup> (1903) describes the use of NPV. Considering and analyzing a house that is offered by a payment of 20 annual installments of 1000 currency units each, the Austrian economist claims that the decision should be made by taking into account the NPV of all 20 installments.

Then, in 1950s, it is suggestive to find two of the earliest references to NPV in the economic literature. Lorie and Savage, dealing with problems of multiple rates of return connected with the internal rate of return method, showed that investment proposals which have positive present value with the firm's cost of capital as the discount factor will also have an internal rate of return greater than the cost of capital.

Although the economists and engineers had made significant contributions to the ideas on discounted cash flows, the accountants had little, if any, impact.

After World War II new developments had occurred and the NPV affect also the accountants. In fact, Johnson and Kaplan<sup>24</sup> (1987) considered the adoption of the NPV rule as the main innovation in management accounting of the last 60 years. Probably the most influential author in popularizing the idea of NPV was the economist and consultant Joel Dean. He wrote the book "*Capital Budgeting*"<sup>25</sup> in 1951 and other articles where he argued that NPV had been used for years but had few applications in capital budgeting.

NPV played a prominent role in the United Shoe Machinery Company antitrust case. The company held more than 90% of the market share in shoe machinery. The District Court of Massachusetts required the company to present a method for calculating a fair price for its machines. It worked with Dean's consulting company, which formulated a pricing scheme based on the NPV rule. This case promoted the use of NPV in American business.

---

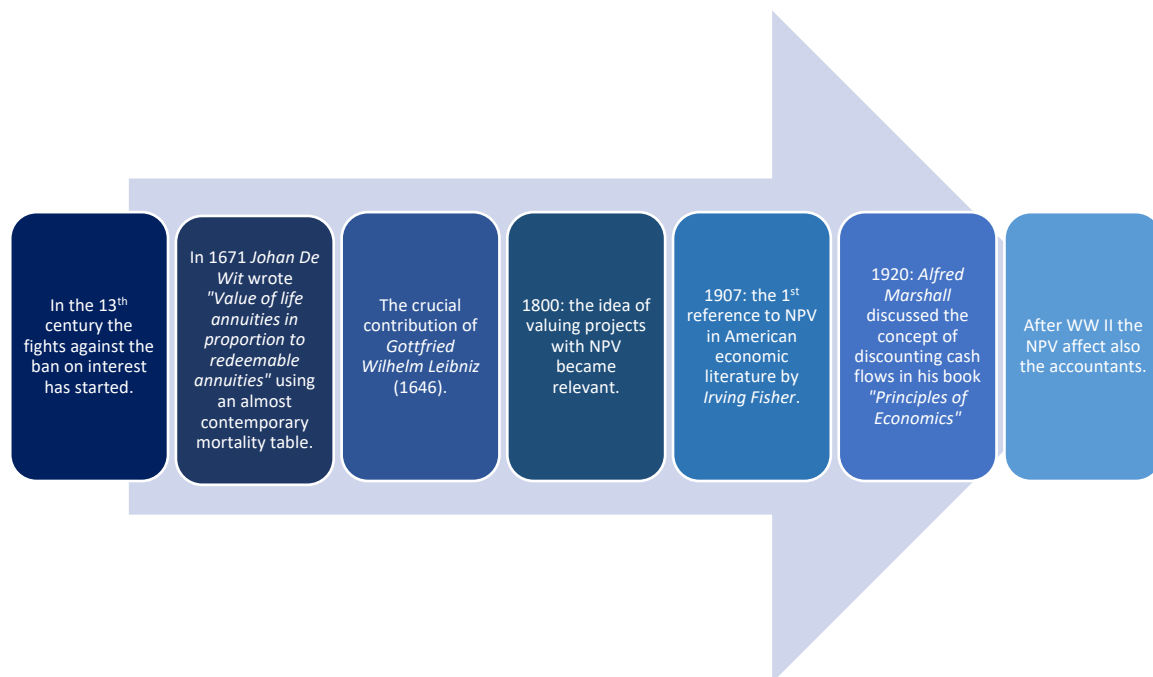
<sup>22</sup> J Behringer (2015), *op. cit.*, p. 81

<sup>23</sup> Behringer (2015), *op. cit.*, p. 81

<sup>24</sup> Behringer (2015), *op. cit.*, p. 82

<sup>25</sup> Behringer (2015), *op. cit.*, p. 82

Not only Dean but also James Lorie and Leonard Savage (1995) with their paper “*Three problems in rationing Capital*”<sup>26</sup> were pivotal in spreading the NPV methodology. Their paper discussed the crucial problem of budget limitation, for instance who have several positive investment opportunities but only a limited amount of money with which to finance these investments. The timeline in fig. 1 depicts the development of the net present value traced above, highlighting the most important events and actors responsible to the method’s evolution.



**Figure 1**

The development of NPV from the 13<sup>th</sup> century until World War II.

### 1.5 *The general rules for applying the Net Present Value rule today*

NPV is currently a broadly accepted rule for valuing projects, appraising investments and specifying the value of the companies. Still, in 1938, Ronald Coase, argued that NPV is the best techniques to make investment decisions.-

The Net Present Value can be seen as how much you need to invest today to produce a certain amount of money tomorrow minus the initial investment.

To invest huge amount of money a company needs to forecast the project’s cash flows and discount them at the opportunity cost of capital to arrive at the project’s NPV. A project with a positive

<sup>26</sup> Behringer (2015), *op. cit.*, p. 83

NPV increases shareholder value and should be accepted. An investment in any asset creates wealth if the discounted value of the future cash flows exceeds the up-front cost. A crucial point is that you have to be aware of what to discount. Therefore, you should stick to four general rules<sup>27</sup>:

1) Only cash flow is relevant.

In this case we are not referring to profits or accounting income. Cash flow is simply the difference between cash received and cash paid out. Therefore, when calculating NPV, it is necessary to record capital expenditure when they occur and not later on when they show up as depreciation otherwise the NPV can result to be negative even if it does not. Depreciation must not be considered because it is not a cash outflow. On the other hand, capital expenditure, which is a proper cash outflow, must be subtract.

It is also important to remember the investment in working capital. As sales increase, the firm may need to make additional investments in working capital and, as the project comes to the end, it will recover those investment so working capital may change during the life of a project. Be also aware of allocated overhead charges such as those for heat or light. These may not reflect the incremental costs of the project.

2) Always estimate cash flows on an incremental basis.

The value of a project depends on all the additional cash flows that follow from project acceptance. Taxes, opportunity costs and all indirect effects of the project, such as its impact on the sales of the firm's other products, need to be included. On the contrary, sunk costs should be ignored because they cannot be affected by the decision to accept or reject the project; they are past and irreversible outflows. The salvage value, net of any taxes, if for instance an equipment is sold at the end of the project's life, represents a positive cash flow to the firm e so they have to be considered. Moreover, financial managers should forecast all incremental cash flows generated by an investment.

3) Be consistent in your treatment of inflation.

If cash flows are forecasted in nominal terms, use a nominal discount rate otherwise use a real one.

4) Separate investment and financing decisions.

---

<sup>27</sup> Brealy, Myers, Allen. 12<sup>th</sup> Edition. *Principles of Corporate Finance*, ch. 6



Forecast cash flows as if the project is all equity financed. Thus, project cash flows should exclude debt interest or the cost of repaying any loans. This is necessary in order to make such a separation.

These principles of valuing capital investment are the same worldwide, but inputs and assumptions vary by country and currency.

Moreover, when choosing between two machines with unequal lives, comparing equivalent annual cash flows is required. Therefore, it is important to calculate equivalent annual cash flows in real terms and adjust for technological change if necessary.

Bodie and Merton argued that “*none of the rules are as universally applicable as the NPV rule*”<sup>28</sup>, as reported by Prof. Dr. Stefan Behringer in his essay. This statement might be the proof that this methodology can be applied also in unusual contexts beyond corporate finance. To confirm this, a case study on the fiscal implications of a newborn in Italy has been conducted below.

---

<sup>28</sup> Behringer (2015), *The Development of the Net Present Value (NPV) Rule – Religious Prohibitions and its Evolution*, p. 83

## CHAPTER II

### A NET PRESENT VALUE CALCULATION: A CASE STUDY ON THE FISCAL IMPLICATIONS OF A NEWBORN IN ITALY

#### 1.1 *Introduction of the case study*

This chapter offer the possibility to examine the net present value fully from another perspective. A case study and the model on which it is based are going to be introduced.

Focusing on the net fiscal effects, the model would like to show the net gain from a newborn in Italy today in a dynamic equilibrium closed economy framework that explicitly accounts for demographics and fiscal policy. Here, the analysis is performed in a simple framework, making several assumptions and simplifications. Nevertheless, the results obtained are pretty clear and offer the opportunity to apply an historical calculation method to the reality. The central “equilibrium” condition is that the government “do not pay” pensions because they are completely offset by the contributions paid by workers and that taxes on labour income adjust so that the current government expenditure policy is sustained over time. The case study is based on the case study “*Fiscal Implications of Immigration – A Net Present Value Calculation*”<sup>29</sup> conducted both in Sweden and in the US<sup>30</sup>. I have built up an easier model. Individual behavior is fixed, as in Lee and Miller<sup>31</sup> (1997), i.e., labour effort, propensities to consume, fertility, employment, probability of surviving are kept constant over time, and wages and interests are exogeneous. Note that each generation bears the same tax burden in the sense that, from the outset, tax rates are constant over time, and wages, government consumption and transfers per person grow at a constant rate. It is important to stress that this is not a welfare analysis but rather a dynamic accounting exercise of government revenues and expenditures through which we can show how the net present value calculation can be exploit even in reality. The model is parameterized using data from Istat, OECD, Inps, and Bank of Italy. Other assumptions are made during the model construction. At the end results will be shown out.

---

<sup>29</sup> Kjetil Storesletten, *Fiscal Implications of Immigration – A Net Present Value Calculation*, Scand. J. of Economics 105(3), 487-506, 2003. University of Oslo, N-0317 Oslo, Norway.

<sup>30</sup> Lee, R. D. and Miller, T. W. (1997), The future Fiscal Impact of Current Immigrants, Chapter 7 in J. P. Smith and B. Edmonton (eds.), *The New Americans*, National Academy Press, Washington, DC

<sup>31</sup> Lee, R. D. and Miller, op. cit.

## 1.2 *Presentation of the model*

The economy in Italy is populated by agents who live up to 100 years. Agents of age  $i$  differ in gender, national origin, labour force status and age. Children of immigrants are assumed to be identical to natives.

Firstly, we want to construct a model upon which we will apply the net present value calculation regarding a new-born in Italy.

Therefore, we would start examining the labour market and the labour force status. Children and retirees do not work. Working-age individuals who goes from 15 to 67 years old, either participate or do not participate in the labour force. If participating, agents are either employed or unemployed and, if employed, agents either work or are on leave (for sickness, rehabilitation or childcare). Note that only those looking for a job are counted as unemployed; those who do not have a job and are not looking for one are counted as not in the labour force. To simplify the calculation, we assume that each agent determines his/her labour-market status for all the future, remaining either permanently out of the labour force or permanently in. Let  $m$  denote the agent's type, consisting of participation status, gender, national origin and age. Earnings for a type  $m$  agent of age  $i$  are given by  $W_t p_{i,m} (1 - u_{i,m})$  where  $W_t$  is the average wage per "efficiency unit" in period  $t$ ,  $p$  the participation rate for type  $m$  of age  $i$ ,  $u$  the unemployment rate for type  $m$  of age  $i$ . Since  $p$  is defined as the ratio of the labour force to the total population of working age and since we have assumed that each agent can be either permanently out or in of the labour force, we can therefore consider  $p$  as fixed over time; while  $W$  is assumed to grow at a constant rate  $z$ .

Proceeding with the model's analysis, now we consider the fiscal policy, which is formulated by the government, and that must be sustainable. Fiscal policy consists of a government consumption rule, which is not essential for the purposes of the NPV calculation, a tax system, and a transfer system, including a pension system. The tax system specifies a constant tax rate on consumption  $t_c$ , the so called VAT or value added tax, a pension contribution tax rate  $t_p$  on earnings, and a tax rate  $t_e(x)$  on taxable earnings, given by

	$t_e = 23\%$	euro 0/15000
	$t_e+4\% = 27\%$	euro 15000/28000
$t_e(x) =$	$t_e+15\% = 38\%$	euro 28000/55000
	$t_e+18\% = 41\%$	euro 55000/75000
	$t_e + 20\% = 43\%$	euro over 75000

We will take an average among these tax rates according to the proportion of the population falling in each gap.

To simplify the transfer system, we assume that there are two types of agents:

- Agents which are unemployed ( $u$ ), between 18 and 67 years old, but are looking for a job taking therefore part of the labour force. Those agents receive work related transfers ( $h_{i,m}$ ) which comprises minimum income guaranteed by the state and cassa integrazione.
- Agents who participate in the labour force ( $p$ ) but are temporary unable and therefore unemployed ( $u$  therefore we consider  $p \times u$ ) for a given period due to maternity leave or sickness, illness or rehabilitation. Those type of agents receives transfer welfare benefits ( $b_{i,m}$ ) for maternity leave, social assistance, social services and family benefits.

Assuming also that the government “do not pay” pensions because they are completely offset by the contributions paid by workers. Therefore, there exists a sort of “equilibrium”. Here we decide to refer to old age pension requirements set in 2018 according to which a private or public employee is entitled to receive a pension after almost 67 years old with a minimum of 20 years of contribution.

In order for the fiscal policy to be sustainable, the government’s intertemporal budget constraint must be satisfied:

$$B_0 = \sum_{t=0}^{\infty} R^{-t}(T_t - G_t - P_t)$$

here  $B_0$  is initial government debt,  $G_t$  is government consumption in period  $t$ ,  $T_t$  is total tax revenue, and  $P_t$  is aggregate transfers. This is the only “equilibrium” restriction of the model.

To compute the net gain for the government of having one extra agent of type  $m$  and age  $I$  at time  $t = 0$ , one must also incorporate the net government gain from the children to whom the agent will give birth. Let  $NPV(0,N)$  denote the net gain from an Italian new-born, where  $N$  stands for natives. This gain must grow at a constant rate  $z$  which represents the growth rate of wages. Since the net gain includes contributions of the agent’s future children, grandchildren, etc., it is given by

$$NPV(0, N) = \sum_{i=0}^{100} \pi_{i,N} R^{-i} (s_{i,N,i} + \phi_{i,N} (1 + z)^i NPV(0, N))$$

$\Leftrightarrow$

$$\text{NPV}(0, N) = \sum_{i=0}^{100} \pi_{i,N} R^{-i} s_{i,N,i} / \left( 1 - \sum_{i=0}^{100} \pi_{i,N} \phi_{i,N} R^{-i} (1+z)^i \right),$$

Where  $\phi_{i,n}$  denotes the annual fertility for an  $i$ -year old native woman,  $\pi_{i,N}$  is the unconditional probability of surviving until age  $i$ ,  $s_{i,m,t}$  denote tax payments less transfers and marginal government expenditures for a type  $i,m$  agent in period  $t$ , given the equilibrium tax rate  $t_e$ ,  $R$  is the gross real interest rate on Italian government debt,  $z$  is the growth rate of wages.

### 1.3 *Data and calculation*

Once the model has been built up, proceeding with the substitution of numbers to variable is quite straightforward. To simplify the model, since we want to calculate the gain obtain from a new-born in Italy today up to 100 years later, we assume that data found in 2019 (or latest data available) keep more or less constant until 2119 according to some forecasts.

The unemployment rate, on a yearly basis considering both women and men, is  $u=10\%$  of the labour force (data from OECD 2019 in Italy). Notice that unemployed people are those who report that they are without work, that they are available for work and that they have taken active steps to find work in the last four weeks.

The labour force participation rate in Italy is  $p=65,5\%$ , taking into account people from 15 to 67 years old (data from OECD 2019 in Italy). Since we have defined earnings for a type  $m$  agent of age  $i$ , it is important also to define  $z$ , the constant rate at which wage per “efficiency unit” in period  $t$  grows. In order to calculate  $z$ , the average growth rate in wages since 1990, we have taken the average of the growth rates of OECD average real annual wages from one year to the subsequent one (from 1990 to 2018 taking 2018 constant prices and NCU). We have taken into account real data not nominal because our goals are to measure wages and their changes over time. Therefore,  $z$  is set equal to  $+0.21\%$ .

Once we have examined the labour market and the labour force status, we can now focus on the fiscal policy.

Value added tax or VAT is a consumption tax charged at a standard rate of  $t_c=22\%$  (4% or 10% only applied to few primary goods).

For what concern the pension contribution tax rate  $t_p$  on earnings, we have taken an average between the contributions paid by independent VAT workers and dependent employees. Starting from the latter the notional account system has a contribution rate of 33% (the rate goes up by only 1% for a gross monthly retribution higher than 3948 euro), of which about one-third is paid by the employee and two-thirds by the employer. At the retirement the pension benefit is calculated as the accumulated lifelong contributions valorized with the nominal GDP growth rate and the transformation coefficient. Even though, benefits are strongly related to retirement age – the lower the age, the lower the pension – we have assumed that each agent who have decided to work, follow this decision along his full working life span until the established retirement age of 67 years old. Considering VAT workers we have taken an average between contributions paid by artisans (23.10%), retailers (23.19%, the rate goes up by only 1% for annual earning higher than 46.123 euro for both artisans and retailers), independent workers without a pension fund (or cassa previdenziale) who would have to pay a contribution rate equal to 25.72% and independent workers (taking into account the major classes and not taking into account special cases for instance young workers who usually pay a lower contribution tax rate). Among those there are lawyers paying 14.5% of subjective contributions according to Cassa Forense, architects and engineers 14.5% according to Inarcassa , notaries an average of 32% according to Cassa Notarile, doctors about 14.5% according to ENPAM, pharmacists 12% according to ENMPAF, psychologists 10% according to ENPAP, accountants a minimum of 12% according to CNPADC. We can now take an average between the two noticing that the independent worker represents about 21.7% of the entire employees (this Istat data has remained more or less the same in the last 10 years). Noticing that data are relative to 2019, we can conclude that the pension contribution tax rate is equal to  $t_p=32.61\%$ .

Data on the income declarations 2018 (tax year 2017), published by the finance department of Mef, shows that about 45% of the population pay a tax rate on taxable earning equal to 23% (0/15000 euro), 50% of the population a tax rate of 27% or 38% (15000/55000 euro) and the remaining part, about 5.3%, a tax rate of 41% or 43% (over 55000 euro). Taking an average, we can assume that the tax rate in taxable earnings is equal to  $t_e=28.86\%$ .

In order to build up the initial government debt is essentially to define work related transfers ( $h_{i,m}$ ) and transfer welfare benefits ( $b_{i,m}$ ). Work related transfers refer to the minimum income guaranteed by the state (or reddito di cittadinanza) which account for 6.1 billion in 2019 (data from Inps) , and unemployment benefits (or cassa integrazione) which, in 2019, amount to 259 billions of hours requested by enterprises. Transfer welfare benefits comprise public spending on incapacity and family benefits spending, both on yearly basis. The former refers to spending due to sickness, disability and occupational injury and it is equal to 1.8% of GDP (OECD data from 2015). The latter, instead, refers to public spending on family benefits, including financial support that is

exclusively for families and children. Broadly speaking there are three types of public spending on family benefits: child-related cash transfers to families with children, public income support payment during periods of parental leave and income support for sole parents' families. All those benefits account for 1.956% of GDP in 2015. The nominal GDP in 2015 is equal to 2,649,820.00 million US dollar or 43,889.00 US dollar/capita.

$\phi_{i,N}$  denotes the annual fertility for an  $i$ -year old native woman according to the following age class in Italy according to the data from Istat 2018:

0-14 years	0%
15-17 years	5%
18-21 years	10%
22-25 years	30%
26-29 years	58%
30-34 years	90%
35-39 years	45%
40-45 years	18%
46-48 years	1%
Over 49 years	0%

The unconditional probability of surviving until age  $i$   $\pi_{i,N}$  for both women and men is shown from the Istat life tables in 2018, according to the following age class:

until 4 years	99.95%
5-9 years	99.96%
10-14 years	99.92%
15-19 years	99.87%
20-24 years	99.84%
25-29 years	99.82%
30-34 years	99.75%
35-39 years	99.62%

40-44 years	99.41%
45-49 years	99.05%
50-54 years	98.49%
55-59 years	97.56%
60-64 years	96.11%
65-69 years	93.87%
70-74 years	89.75%
75-79 years	82.56%
80-84 years	68.92%
85-89 years	49.39%
90-94 years	29.52%
95-99 years	16.57%
100 years	5.12%

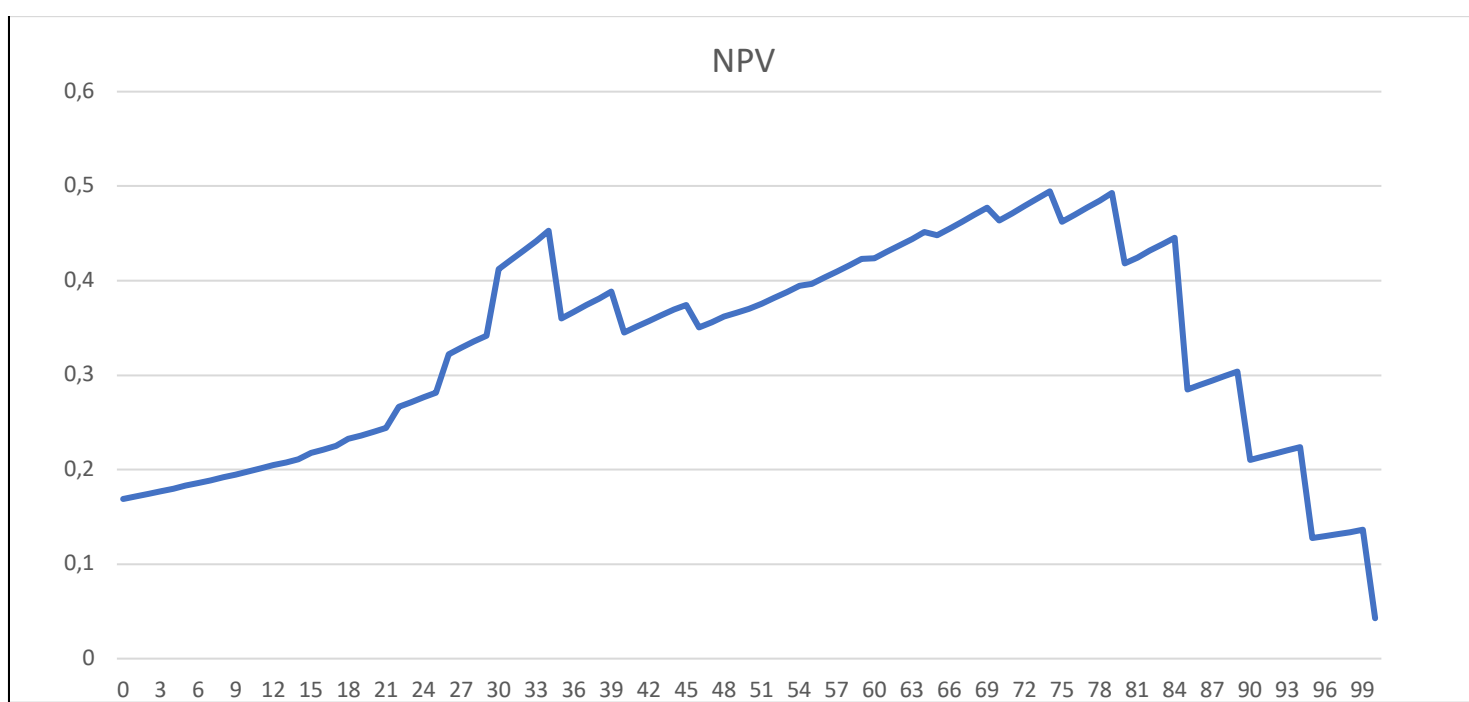
Lastly, we consider a gross real interest rate on Italian government debt  $R$  equal to 1.0161 (implicit average interest or “interesse medio implicito” in 2019)

#### 1.4 *Results and discussions*

Figure 2 shows the discounted net government gain from an Italian new-born from age 0 to 100 years in terms of percentage. It is noticeable that, from the beginning of their working years (15), the line starts to slope upward meaning that natives bring surplus to government coffers, albeit with some relapses. Not surprisingly, when young (1-14) and when old (67+), natives, even if they are not properly net burdens, represent a much little gain for the government. This is confirmed by the fact that the NPV become to fall from 74 years old until reaching the lowest value of 0.0427 (equal for all the three figures since the value is discounted by  $R$  at the power of 0) at the age of 100. The expected net gain of a new-born in Italy is euro +33.1656. The positive net gain of a new-born confirms the sustainability of the fiscal policy.



In the comparable study cited above<sup>32</sup> and in a similar one conducted in US by Lee and Miller<sup>33</sup> (1997), the average net government gain from new immigrants are higher, in particular for US, than the figures for Italy reported here. The main reasons for this large discrepancy are that both US and Sweden have a larger government sector, higher tax rates, more redistribution to non-working individuals, higher wages, participation rate and gross real interest rate. Moreover, the case study in question has been built on a simpler model. Nevertheless, the results obtained appear to be in line with previous expectations: people out of labour force (i.e. young and old people) contributes much less to the government gain with respect to an individual during its working age (15-67). Although, children and retirees, who do not work, represent a government expenditures, their NPV will not turn out to be negative also because of the existence of the simplified model's central "equilibrium" condition : the government "do not pay" pensions because they are completely offset by the contributions paid by workers and that taxes on labour income adjust so that the current government expenditure policy is sustained over time. In this way taxes are higher than government benefits and this led to a positive NPV but strictly close to zero confirming the fact that they do not contribute, yet or anymore, to the welfare of the government.



**Figure 2**

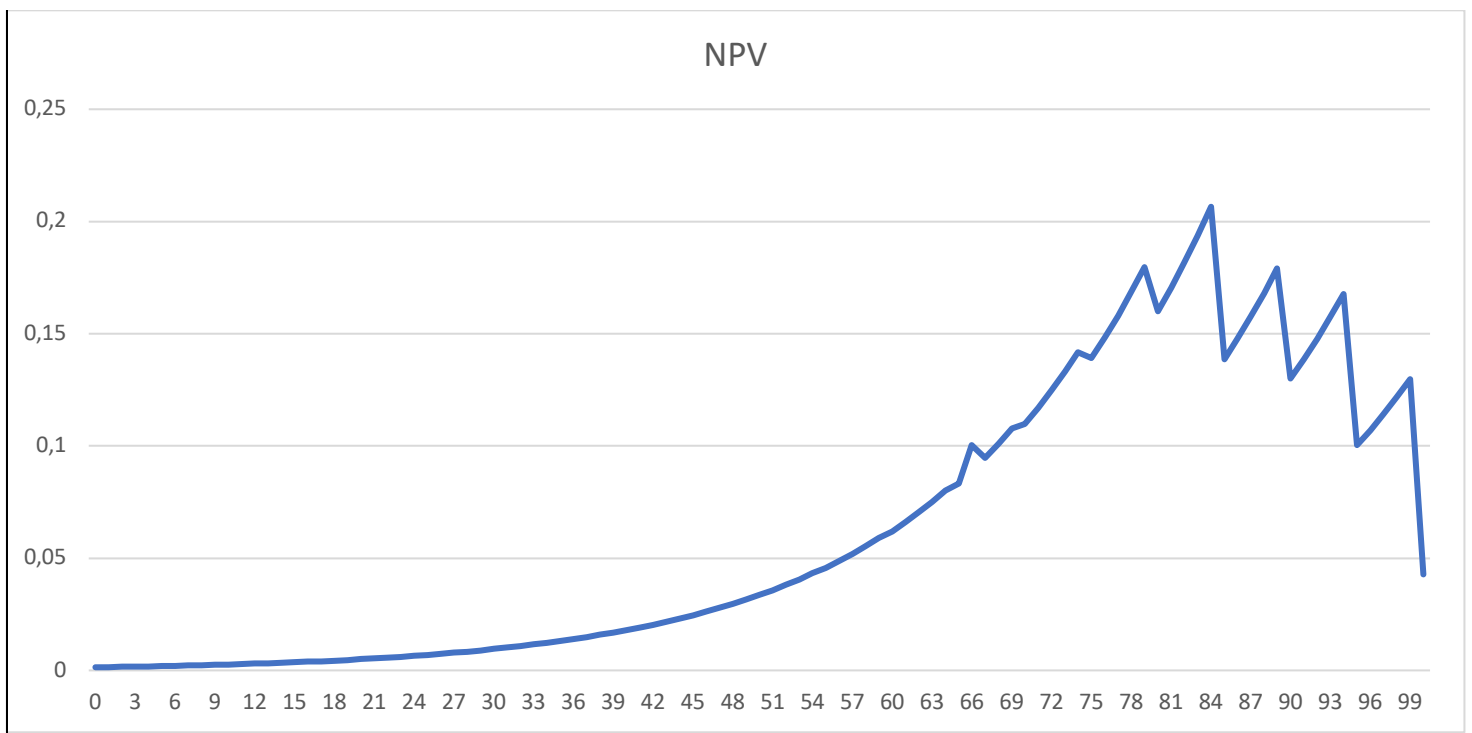
Calculation of NPV of a newborn in Italy today from age 0 to age 100 discounted at the gross real interest rate on Italian government debt  $R=1.0161$

<sup>32</sup> Kjetil Storesletten, op. cit.

<sup>33</sup> Lee, R. D. and Miller, op. cit.

So far, I have assumed that the gross real interest rate on Italian government debt  $R$  is equal to 1.0161. The results are quite sensitive to this rate. In fact, changing the rate the results will change as well. The graphs below show two alternative and opposite cases: an increase in  $R$  by 5% (fig. 2) and a decrease in  $R$  by 5% (fig. 3).

Firstly, I consider the effects of an increase in  $R$  by 0.05, therefore I take into consideration a discount rate equal to  $R=1.0661$ . Figure 3 below shows that the NPV of an Italian new-born is nearly zero from age of 0 until age of 20. Then it starts to soar year after year, reaching its peak between 82-84 years. Although with some fluctuations, the NPV begins to decrease from age of about 84 years old. With respect to the graph above, the lowest value has reached at 0 year not at 100 years old. With the slight increase of  $R$ , the curve of the figure under consideration seems to be shifted to the right with respect to Fig. 1. In this case we obtain a positive total NPV throughout the entire life span of an individual lower than before and equal to euro +6.2770.



**Figure 3**

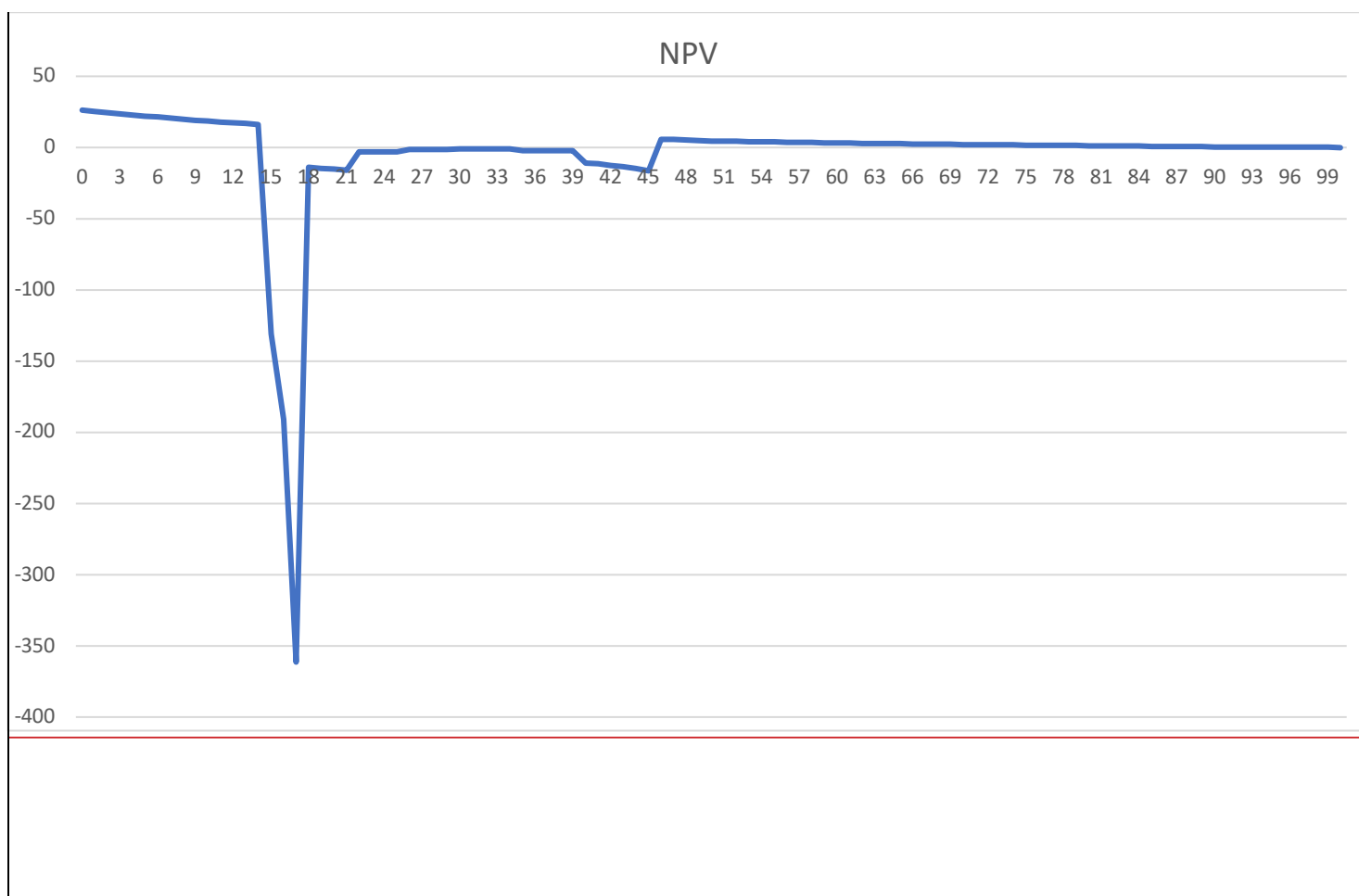
Calculation of NPV of a newborn in Italy today from age 0 to age 100 after an increase of the discount rate by 5% with  $R=1.0661$

Then I consider the opposite case: figure 4 below describes a decrease in  $R$  by 5%. Thus, this time the discount rate will become  $R=0.9661$ . From an economic point of view the smaller the gross real interest

rate on Italian government debt the lower is the debt-to-GDP ratio which measures the ability of a country to pay back its debts and therefore its performance. We can clearly see that the NPV is quite constant and positive during the first 14 years and then it dramatically decreases, becoming negative until age of 45. It reaches its minimum value more or less at age of 17 years old. After that it sharply increases and becomes again positive. At this point it starts decreasing until reaching the usual common value of 0.0427 at age of 100.

In the following case we obtain a curve and subsequent results which are really unlike with respect those previously obtained. Here the total NPV is negative and equal to euro -426.59017.

Therefore, we can conclude that, the lower the gross real interest rate on Italian government debt the smaller the net gain obtained by the government from a new-born.



**Figure 4**

Calculation of NPV of a newborn in Italy today from age 0 to age 100 after a decrease of the discount rate by 5% with  $R=0.9661$

The decrease between 15 and 45 years old can be explained by the fact that the State invests a lot in a person who is entering the labour market and during its first 20 years of job career. This decrease is probably emphasized by the difference, as we have said before, with the case studied by Lee and

Miller<sup>34</sup> and that one described by Storesletten<sup>35</sup>, according to which both countries in question, US and Sweden, have a larger government sector, higher tax rates, more redistribution to non-working individuals, higher wages, participation rate and gross real interest rate. All these factors could explain the prominent and dramatic decrease between 15 and 45 years old.

---

<sup>34</sup> Lee, R. D. and Miller, op. cit.

<sup>35</sup> Kjetil Storesletten, op. cit.

## CONCLUSION

The Net Present Value methodology was not used in common practice before the 1960s; it needed the practical approach of consultants for its breakthrough. It has long been regarded in academic circles as the best criterion for investment appraisal; however, several alternatives and complementary methods remain popular with practitioners. Moreover, several of these standard criteria, such as the net final value, benefit-cost ratio, profitability index, equivalent annuity, discounted payback [...], lead to the same results and therefore investment decisions if properly applied. Another important point studied by authors was to analyse whenever NPV is better than Internal Rate of Return (IRR) as NPV is calculated on capital cost and IRR is determined on calculated IRR rate. After a great variety of quantitative studies were conducted, authors decide to take the stock about the possible better performance of NPV. It has been found that for individual projects IRR is used mostly and NPV is preferable when the projects are mutually exclusive. But sometimes investors prefer to use NPV because it is easy to calculate and reinvest the cash flows at the cost of capital. Nevertheless, sometimes IRR is preferable because it gives answer in percentage and it is easy to understand even if IRR reinvest at calculated IRR. Albeit the question is quite controversial most authors agree upon the hypothesis that NPV is better than IRR.

All these multiple reasons, as shown, have made the development of NPV really slow but they also show out the simplicity and efficacy of this methodology.

Today, NPV is a key concept in finance and management. It is widely accepted in theory and practice even if it has a much tricky history with respect to other globally accepted principles. This is caused by the ban of interest triggered in religious word and in particular on the compound interest which, furthermore, had resulted in the delay of developing further banking and management instruments.

Throughout the case study, aimed at examining the fiscal implications of a new-born in Italy, it can be seen how an ancient methodology can be applied beyond the usual “bounds” of corporate finance. Albeit several simplifications and assumptions have been made, the results obtained are in line with previous expectations. The discounted government gain from an Italian new-born was computed using a simple net present value calculation, taking into account the Italian tax system, government expenditures and fiscal policy. The model shows possible gains from new-borns. This happens particularly during natives’ working age (15-67) because children and retirees, who do not work, represent a major “cash out” for the government. It can be said that the NPV of an Italian new-born is larger than zero.

## REFERENCES

- Arma Arshad. 2012. “*Net Present Value is better than Internal Rate of Return*”. Interdisciplinary journal of contemporary research in business. University of Faisalabad.
- Armen A. Alchian. 1995. “*The Rate of Interest, Fisher’s Rate of Return over Costs and Keynes’s Internal Rate of Return*”. American Economic Association. The American Economic Review, Vol. 45. 45, No. 5, pp 938-943.
- Brealy, Myers, Allen. 12<sup>th</sup> Edition. “*Principles of Corporate Finance*”.
- Donald S. Remer, Armando P. Nieto. 1995. A compendium and comparison of 25 project evaluation techniques. “*Part 1: Net present value and rate of return methods*”. Harvey Mudd College of Engineering and Science.
- Joan Pasqual, Emilio Padilla, Evans Jadotte. 2012. “*Technical Note: Equivalence of different profitability criteria with the net present value*”. Int. J. Production Economics. University Autonoma de Barcelona.
- Kjetil Storesletten. 2003. “*Fiscal Implications of Immigration – A Net Present Value Calculation*”. Scand. J. of Economics 105(3), 487-506, 2003. University of Oslo, N-0317 Oslo, Norway.
- Lee, R. D. and Miller, T. W.. 1997. “*The future Fiscal Impact of Current Immigrants*”. Chapter 7 in J. P. Smith and B. Edmonton (eds.), *The New Americans*, National Academy Press, Washington, DC
- Michael C. Jensen, Clifford W Smith. 1984. “*The Theory of Corporate Finance: A Historical Overview*”. Harvard Business School, University of Rochester. The modern theory of corporate finance, New York: McGraw-Hill Inc., pp. 2-20.
- Stefan Behringer. 2015. “*The Development of the Net Present Value (NPV) Rule – Religious Prohibitions and Its Evolution*”. Review of Economics & Finance. University of Applied Sciences Kolner Chaussee.
- Stephen A. Ross. 1995. “*Uses, Abuses, and Alternatives to the Net-Present-Value Rule*”. Wiley on behalf of the Financial Management Association International. Financial Management, Vol. 24, No. 3, pp. 96-102.
- Thomas W. Jones and T. David Smith. 1982. “*An historical perspective of Net Present Value and Equivalent Annual Cost*”. The Academy of Accounting Historians. The Accounting Historians Journal, Vol. 9, No. 1, pp. 103-110.