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Empirical evidence on PPP deviations' persistence

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Abstract

In this dissertation we tackle the issue of Real exchange rate convergence towards Purchasing Power Parity (PPP). The motivation of such analysis stems from the remarkable importance that such no-arbitrage condition has in macroeconomics, both in a policy design framework and in a foreign exchange market environment. The aim of the thesis is twofold: firstly, we will analyse globally PPP deviations' persistence, trying to spot significant differences among different categories of countries; the second part of the analysis focuses instead on a technicality about the real exchange rate components: we try to evaluate the hypothesis of employing an alternative price index different from the typically used CPI, which takes into consideration price fluctuations of several categories of goods. The first objective is achieved by fitting both linear autoregressive and nonlinear smooth-transition models to the real exchange rate series and therefore computing real rates' half-lives, which we adopt as a metrics of persistence, via model parameters or through impulse response functions. We apply instead a Principal component analysis to several price index series to estimate a core price index as the series' common factor, which is then employed in real exchange rate computation. While on the one hand we only get few significant results in the global analysis, we manage to show that in most countries the factor-adjusted real exchange rate tends to be way less persistent than its standard CPI counterpart, both in a linear and nonlinear fashion.

Summary

1 Introduction

The theory of Purchasing Power Parity (PPP) constitutes a central pillar in the context of modelling relationships among countries. The stability of real exchange rates, combined with their capability of converging after a shock is a crucial issue to be considered when studying macroeconomic policies. This is true both in the case of an exchange rate regime decision and in the broader framework of price stability-oriented monetary policy.

The aspect this thesis aims to contribute on is the persistence of deviations from PPP. From Benigno (2004), we learn that alternative monetary policy measures might have different effects on real exchange rates, leading to more or less persistent deviations from PPP, even without excessively burdening menu costs. The natural consequence is an impact on a country competitiveness, as the real exchange rate plays an important role in output determination, for example, in a Mundell-Fleming context: if a contractionary policy is put in place, a higher interest rate will lead to currency appreciation and therefore to an increase in the real exchange rate; then, exports will be more costly for foreigners, but domestic imports will be cheaper, implying an improved trading position. If prices are sufficiently sticky, this can be easily viewed as a real exchange rate shock. Thus, knowing how persistent in the future such shock will be, helps the authority to optimally calibrate policy measures by internalising such information.

Also nominal exchange rate regime decisions play a role in this framework: highly volatile nominal rates in a floating regime might induce policymakers to be more careful in implementing measures which are likely to cause higher uncertainty, especially with high menu costs. Moreover, nominal exchange rates forecasting itself would benefit from a comprehensive knowledge of real exchange rate persistence, given the fact that often their movements are consequences of PPP divergences; this has clearly an impact not only on the real economy, but also on financial markets.

The first idea we apply to study real exchange rates entails analysing a multitude of countries with the objective of spotting any potentially significant difference in PPP deviations' persistence according to different country categorizations. Using a similar approach to the one adopted in Curran and Velic (2019), this task is be accomplished by fitting time-series models to bilateral and multilateral (effective) real exchange rates. We also provide an insight on the question of whether nonlinear models tend to perform better than linear ones in approximating real exchange rates behaviour, yielding, as a consequence, different persistence results.

The innovative contribution to the existing literature that we attempt to provide in this thesis lays in the construction method of the bilateral exchange rate: we try to implement a principal component analysis which serves as a way to obtain the *true* (unobserved) price index of a given country, which should not be subject to the limitations of the standard consumer price index which is typically employed in the real rates construction process.

Our findings do not confirm the presence of significant differences in PPP deviations' persistence in most cases; nevertheless, we are able to corroborate the hypothesis that substantial differences emerge from using either of the two price indexes described. Also the linear-nonlinear model comparison yields interesting results: in most cases, half-lives estimated through nonlinear models tend to be clearly lower than the ones obtained through linear ones. The rest of the thesis is organised as follows: in section 2 we will present a review of the literature on the topics of real exchange rate and purchasing power parity, with a focus on the concept of half-life (employed as a measure of persistence). Section 3 is devoted to the description of the models employed for half-lives calculation, together with a presentation of the factor extraction process via principal components. In section 4 we present our data and data sources, while in section 5 we show the main empirical results. Section 6 concludes the dissertation.

2 Literature Review

2.1 Early studies

As Froot and Rogoff (1995) pointed out in their broad review of the literature on PPP studies, analytical methods have evolved with time; it is thus useful pointing out the most interesting hypothesis on the presence of possible short-term PPP misalignments.

To begin, we must recognise that often real exchange rate series follow a unit root process and therefore do not show any significant feature of mean reversion; nevertheless, sometimes it is possible to investigate instead the possibility that despite the fact that real exchange rates may deviate from unity in the short term, there might be a long run relationship among its components (nominal exchange rates and prices). Such hypothesis of cointegration among nominal exchange rates and price index series has been studied, among others by Enders (1988) and Corbae and Ouliaris (1988).

2.2 PPP failure and the Balassa-Samuelson effect

Reasons why PPP might fail can be largely diverse. One of the main arguments comes from the Balassa-Samuelson effect, theorized in Balassa (1964) and Samuelson (1964): it attributes the deviation from PPP to the presence of non-tradable goods, whose prices cannot adjust through trade arbitrage. As a consequence, more productive countries tend to have higher prices for non-tradable goods, since the higher marginal products correspond to higher labour costs and hence to higher prices.

2.3 New approaches and nonlinearities

In the more recent literature, it is not uncommon to investigate the determinants of real exchange rate movements; One example of such analysis can be found in Bergin, Glick, Wu (2017), where the focus is on "Conditional PPP", that is a measurement of the speed of convergence to PPP depending on different sources of shocks and on different adjustment mechanisms (prices and/or nominal exchange rates).

In Ca' Zorzi and Rubaszek (2020) this kind of analysis is exploited in a forecasting framework. One interesting finding of this study is that a calibrated model not only beats the random walk in terms of forecasting error, but also other more complex model specifications, including nonlinear models.

A DSGE model is adopted in Rebelo, Eichenbaum, Johannsen (2019), where it is stated that nominal exchange rates are predictable *through* real rates. As in Ca' Zorzi and Rubaszek (2020), nominal rates account as the major of real exchange rates movements.

The widespread idea that real rates follow a random walk has been questioned, among others, in Kilian and Taylor (2003), who state that due to the presence of nonlinearities in the data, simple autoregressive models may not be enough to precisely capture real rates dynamics. What they propose is to apply an Exponential Smooth Transition Autoregressive Model (ESTAR). In the methodological section we will have a closer look at this kind of model, which accounts for different behaviours depending on the closeness of the series to the PPP mean.

2.4 PPP deviations and their persistence: the role of Half-Lives

As we mentioned above, the main focus of this study is on the presence of Purchasing Power Parity; we have seen that various attempts have been made to question the statement that, since real exchange rates follow a random walk, Purchasing Power Parity is unfeasible at all time horizons. What we aim to do here is to propose a slightly different approach, which follows the strand of literature which focuses on the duration of purchasing power parity failure. One of the most interesting studies in this direction is Kilian and Zha (2002), in which they used a bayesian framework to study the degree of mean reversion of the real exchange rate through the Half-Life (HL) of PPP deviations following a shock in the real exchange rate. They justify this kind of analysis stating that the rejection of the null of unit root in real rates series is not enough to guarantee a sufficiently high degree of mean reversion that is compatible with the presence of PPP.

2.4.1 Global analysis

Among the papers that inspired this thesis and on the basis of which our work is conducted is undoubtedly Curran and Velic (2019). This recent study contains a global analysis of the persistence of PPP deviations after a shock. The huge number of countries involved in this study allows the authors to distinguish countries according to different criteria, and then to compare the results among the different categorizations. One peculiarity of this study is the use of the ESTAR model as we saw in Kilian and Taylor (2003). They adopted a very rigorous approach performing the linearity test proposed by Teräsvirta (1994), which constitutes one of the main building blocks of the strand of literature which studies series' nonlinearity, and in particular STAR models. Once again, the object under observation for understanding the persistence of PPP deviations is the half-life. The half-life is a commonly used tool in the field of international macroeconomics; we will adopt the following half-life definition: *"The half-life of the real exchange rate process is commonly defined as the number of years required for the impulse response to a unit shock to dissipate by one half"* [1].

Curran and Velic (2019) is not the only example of global analysis. Cheung and Lai (2000a) also choose more than 90 countries to conduct an analysis which would include not only industrialised countries but also nations which are considered developing or outside the high-income group. This has been justified by the authors by arguing that taking into consideration only industrialised countries might be a source of bias in concluding either in favour or against PPP reversion. Among the main argument, we recall the above-mentioned Balassa-Samuelson effect, which mainly characterises advanced economies.

2.5 Beyond the Consumer Price Index

What this thesis differs in mostly from the above-mentioned studies is the idea that one source of bias could come from the measurement of the price index which is used to construct real exchange rates. Dow (1994) clearly states that there is a certain probability that the commonly used Consumer Price Index might suffer from a series of biases. In particular, according to this paper the CPI might be misleading in the fact that it incorporates prices of goods which, if subject to a shock, can cause distortions in the CPI itself. The author argues that attempting to solve this problem by using indexes which exclude these kinds of goods (e.g. Food, Energy) would cause

[1] Kilian and Zha (2002).

the loss of information coming from such goods' prices. Even before Dow (1994), in Bryan and Cecchetti (1993) the issue of weighting goods when constructing the CPI is raised. In particular, it is explained that inflation as measured by the CPI is a noisy counterpart of the underlying common price increase factor (which, in reality, is unobserved). Such discrepancy might come either from measurement error (on which little can be done) or from weighting bias, as announced before. Although they apply this reasoning in a different context than the one of real exchange rates (i.e. monetary policy) we believe that these arguments hold also in our context. Dow (1994) suggests to employ a filtered version of the CPI: through the use of the Kalman filter, it is possible to obtain an index which attempts to summarize information coming from price index series other from the All-Items CPI. A second option is the use of a factor approach; following Stock and Watson (2010) it is possible to implement a Principal Component Analysis (PCA) in order to extract one factor that summarizes the information contained in a multitude of price index series. The main advantage of this approach, compared to the Kalman filter, is that it rules out the potential bias coming from the estimation of the state-space model coefficients and, most importantly, from the prior choice. Through PCA we are therefore agnostic about the behavior of the series we extract from all the price index series at our disposal. Examples of this approach can be found in Maria (2004) or Machado *et al.* (2001), which both propose to use factors to estimate the *core* inflation through the extraction of the first principal component from a series of CPI items. In particular, the former suggests an OLS-adjusted version of the *core* inflation series, that means using as the reference price changes indicator the series composed by the fitted values of the OLS regression of the standard CPI on the first estimated principal component. According to Maria (2004) this approach is likely to improve the findings of Machado *et al.* (2001) on a reference unbiased price index .

Therefore, in addition to the analysis of half-lives across countries, we will contribute to the existing literature by providing a parallel estimation of time series models and half-lives calculation for a subset of countries using an alternative price index constructed via principal component. We will therefore be able to compare the results of this estimation with the ones obtained using standard CPI for the countries involved.

3 Models

The first object we will use in our analysis is the Bilateral Real Exchange Rate (BRER) with respect the the United States (which we take as the "home" country). The choice of the US as the reference country is quite common in the literature. In particular, we will define the BRER as the price of US goods in terms of foreign goods; thus, it is constructed as follows:

$$BRER_t = e_t + p_t - p_t^* \tag{1}$$

where each variable is in natural logarithm (that is why variables are added and subtracted and not multiplied). e_t is the nominal exchange rate, defined as the price of one dollar in terms of foreign currency (e.g. £/$ for UK); p_t is the US price index, while p_t^* is the foreign country price index (for the main analysis of this thesis, the CPI (Consumer Price Index) will be used as a proxy for a country's price level). In terms of appreciation and depreciation, from this definition of the real rate we get that an increase in the BRER value implies an appreciation of US goods relatively to the foreign ones. This means that US exports become relatively more expensive in terms of foreign goods. It is crucial to underline one important aspect: the BRER we are going to analyse, being constructed as a combination of indexes, is going an index itself. Therefore, it cannot be directly employed to observe absolute PPP in a specific period, but it can still be used to observe PPP convergence across countries. Such convergence (or better, speed of convergence) will be measured through real exchange rates' half lives. The same thing will be done using the Real Effective Exchange Rate (REER). Such index, differently from the BRER, measures a country trade competitiveness not only against another single country, but against many, being a weighted average of several bilateral rates. As for the BRER, an increase in the REER of a given country implies an appreciation of that country's exports, and therefore a weakening of its trade position internationally. From now on, we will refer to both rates using the abbreviation "RER".

3.1 AR(1) Model

In order to compute half-lives, we will fit different models to our univariate RER series. To begin with, we apply the following model specification (define $r_t = RER_t$):

$$r_t = \alpha + \rho r_{t-1} + \varepsilon_t \tag{2}$$

This simple $AR(1)$ model allows us to compute the country half-life either directly from its coefficient or through the impulse response function. Recall that the half-life is defined as the number of years it takes for a unit shock to be be absorbed by one half. Therefore, the latter method works as follows: assuming that the shock hits at time t and defining $I(t)$ as the value of the relative impulse response function, the half-life for a given model is defined as $h \ s.t. \ I(t+h) \approx 0.5$ and

$$\begin{cases} I(t+h+k) < 0.5 \\ I(t+h+k) < I(t+h) \end{cases}$$

for $k = 1, 2, 3...$

In this case the IRF will take the form $I(t) = \rho^t$.

The closed formula for half-life calculation in an $AR(1)$ model, following Curran and Velic (2019) is: [2]

$$\hat{h}_{AR(1)} = \frac{log(0.5)}{log(\hat{\rho})} \tag{3}$$

Although straightforward, this model is quite important; in the literature we saw that the question of whether the RER series exhibit a unit root is still open. Therefore, by fitting this kind of model we are able to have a close look at this issue. We expect in fact a value of $\hat{\rho}$ close but below unity. In this way, we would obtain a highly persistent process but with a potential mean reversion tendency towards PPP, which would imply a finite value for the relative half-life.

3.2 Augmented AR model with linear trend

Due to the trending feature of the RER series, we apply an augmented model in which we add up to twelve lags and a linear trend component. This model specification is presented in an error correction form: we actually regress the *differential* of the RER at time t on the RER *value* at $t-1$ and on the lagged differentials. The linear trend is denoted by τ:

$$\Delta r_t = \beta_0 + \mu\tau + \rho r_{t-1} + \sum_{j=1}^{p} \beta_j \Delta r_{t-j} + \varepsilon_t \tag{4}$$

p is the number of lags.[3] With this kind of model we compute the half-life through

[2]"log" stands for the natural logarithm.

[3]$p_{max} = 12$. Kilian and Zah (2002) use 12 lags for monthly data. We select the optimal number of lags through the Bayesian information criterion (BIC).

the impulse response function as we have seen above. Note that now, being a higher-order model, the IRF will be given by the (1,1) element of the q x q matrix F, where $q = p + 1$ and represents the order of the autoregressive model. The matrix F takes the following form:

$$F = \begin{bmatrix} 1+\rho+\beta_1 & \beta_2-\beta_1 & \dots & \beta_p-\beta_{p-1} & -\beta_p \\ 1 & 0 & \dots & 0 & 0 \\ 0 & 1 & \dots & 0 & 0 \\ \vdots & 0 & \ddots & 0 & \vdots \\ 0 & 0 & \dots & 1 & 0 \end{bmatrix}$$

Identifying the upper-left element of F as $f_{1,1}$ the impulse response to be analysed for the half-life computation is $I(t) = (f_{1,1})^t$. The half life will be equal to h *s.t.* $(f_{1,1})^{t+h} \approx 0.5$. Note that here we are not specifying a particular kind of shock, but we refer to a generic one. As Curran and Velic clarify indeed, there should not be any kind of shock with a persistent effect on the real exchange rate, provided that it follows a mean-reverting process.

3.3 ESTAR model

As anticipated before, linear models are not the only ones we will fit to the RER series. In particular, among the group of nonlinear models we will consider an Exponential Smooth Transition Autoregressive Model (ESTAR). The main feature of this model is the fact that the transition between regimes occurs through an inverse exponential function, which takes the form of a U-shaped curve. The function ranges between zero and one: when the series is close to a certain threshold value, the function takes value zero, otherwise it takes value one. The speed of the transition between the two regimes is determined by a specific parameter which captures the smoothness of the curve. In the context of our analysis, this turns out to be useful, as it has a straightforward economic interpretation: the basic intuition is that, depending on the distance of real exchange rate from its PPP value, it might have a different behavior, and again the transition between the two regimes might not necessarily be linear.

The mathematical formulation of such model is as follows:

$$\Delta r_t = \alpha + \rho r_{t-1} + \sum_{j=1}^{p} \beta_j \Delta r_{t-j} + G(\gamma, th, r_{t-1})(\alpha^* + \rho^* r_{t-1} + \sum_{j=1}^{p} \beta_j^* \Delta r_{t-j}) + \varepsilon_t \quad (5)$$

The function $G(\gamma, th, r_{t-1})$ is the (inverse) exponential function, and takes the following form:

$$G(\gamma, th, r_{t-1}) = 1 - exp(-\gamma(r_{t-1} - th)^2) \quad (6)$$

Where γ is the smoothness parameter while th is the estimated threshold.

The different behaviour that the series might take is therefore determined by the value of the above function. In particular, when the lagged RER series is close to its PPP level ($r_{t-1} \approx th$) and if $\gamma > 0$ the model becomes:

$$\Delta r_t = \alpha + \rho r_{t-1} + \sum_{j=1}^{p} \beta_j \Delta r_{t-j} + \varepsilon_t \quad (7)$$

Which is an "error correction" version of a simple AR(q)[4] linear model. Otherwise, if the function $G(\gamma, th, r_{t-1})$ takes value 1, which implies that the term $(r_{t-1} - th)$ is approaching infinity, the "full" model specification is:

[4] Recall: $q = p + 1$, where p is the number of lags in the equation.

$$\Delta r_t = (\alpha + \alpha^*) + (\rho + \rho^*) r_{t-1} + \sum_{j=1}^{p} (\beta_j + \beta_j^*) \Delta r_{t-j} + \varepsilon_t \tag{8}$$

As in the $AR(1)$ model, we can employ a formula to compute the half-life using the ESTAR model parameters:

$$\hat{h}_{ESTAR} = \frac{log(0.5)}{log(1 + \hat{\rho} * (1 - exp(-\gamma(r_{t-1} - th)^2)))} \tag{9}$$

Following Curran and Velic (2019), the transition function for the computation of the half-life will be evaluated both at its maximum and at its mean value.

3.4 Principal Component Analysis

In the introduction and in the section on the literature review about real exchange rate analysis, we mentioned the issue of potential biases from which the CPI might be affected. Adding the fact that the *true* price index of a given country is not indeed observed, a suitable correction in this sense seems to be needed. For this reason, in a parallel way with respect to the analysis described so far, we will try to fit the above-described models to the bilateral real exchange rate series constructed using an estimation of the price index of given countries obtained through PCA (Principal Component Analysis) instead of the standard CPI. The objective is, on the one hand, to obtain and index that summarizes all the information contained in a series of indexes which track the price level of different categories of goods and services; on the other hand, at the same time, this estimated index should be robust enough not to be excessively influenced either from distortions coming from shocks which affect specific goods' prices or from measurement and weighting errors in which it is possible to incur when constructing the CPI.

From a technical point of view, we will try to extract the first principal component from a series of stationary time-series which capture the inflation rate in different categories of goods, which are described in the appendix[5]. In order to do so, we apply the typical PCA procedure described in Stock and Watson (2010), which we illustrate below.

We start from the equation that links the data to the factors and their loadings:

$$X_t = \Lambda F_t + \varepsilon_t \tag{10}$$

where X_t is the matrix containing our data, F_t is the matrix of the factors to extract (or vector, in case the factor to be extracted is just one), and Λ is the matrix of factor loadings. Factors and factor loadings are then jointly estimated through the following minimization procedure:

$$\begin{aligned} &\min_{F_t,\Lambda}(NT)^{-1}\sum_{t=1}^{T}(X_t - \Lambda F_t)'(X_t - \Lambda F_t) \\ &s.t. \;\; N^{-1}(\Lambda'\Lambda) = I \end{aligned} \tag{11}$$

Such optimization can be carried out in two steps: first, taking the FOC over F_t, we obtain $\hat{F}_t = (\Lambda'\Lambda)^{-1}\Lambda' X_t$. Plugging this result in the objective function we obtain that the minimization problem becomes:

$$\begin{aligned} &\min_{\Lambda}(T)^{-1}\sum_{t=1}^{T}X_t'(\Lambda(\Lambda'\Lambda)^{-1}\Lambda)X_t \\ &s.t. \;\; N^{-1}(\Lambda'\Lambda) = I \end{aligned} \tag{12}$$

[5]Inflation series are scaled and centered, so that they have mean zero and unit variance.

The solution to this problem is found setting the matrix of the estimated factor loadings $\hat{\Lambda}$ to be equal to the matrix composed by the eigenvectors corresponding to the q[6] largest eigenvalues of the variance-covariance matrix of X_t, $\hat{\Sigma}_{XX}$. What we need to underline here is the normalization $N^{-1}(\Lambda'\Lambda) = I$; without this assumption, the matrices $\hat{F}_t$ and $\hat{\Lambda}$ would be unique up to a scalar. This would happen because if we replaced Λ with $\Lambda = \Lambda H^{-1}$ and F_t with $F_t = F_t H$, where H is a nonsingular matrix, the variance of the data $\Sigma_{XX} = \Lambda\Sigma_F\Lambda'$ would be the same, and therefore $\hat{F}_t$ and $\hat{\Lambda}$ would not be identified. Having said that, we can finally see that the matrix (vector) of principal components is:

$$\hat{F}_t = N^{-1}\hat{\Lambda}'X_t \tag{13}$$

4 Data

4.1 Data sources

Data on effective exchange rates have been directly retrieved from the International Monetary Fund's *International Financial Statistics* (IFS). As briefly mentioned before, such rates are the result of a weighted averaging procedure carried out by the IMF itself. In a note of March 26^{th} 2019, the IMF specified that available data were collected in collaboration with institutions such as the Bank for International Settlements and the European Central Bank, that the pool of countries involved in the calculations has been increased to 31 countries, and that since 2004 weights have been updated every three years. The exact procedure of REER calculation is described in Bayoumi *et al.* (2005). Bilateral rates series, instead, have been constructed manually using nominal exchange rates and price indexes. As for the REERs, data

[6] q is the number of factors we want to extract.

on nominal rates and on CPIs were taken from IMF's IFS. BRERs data have been collected for a total of 61 countries, while data on REERs were available for a total of 44 countries. The sample period taken into consideration for the majority of the countries in the BRER analysis starts in 1970 to end in 2020. There are some isolated exceptions with less data available, in particular some of the former USSR countries have data available starting from 1992. Data on REERs start instead from 1980, to end in 2020 as well. All data are monthly period averages. As announced before, we will be able to compare persistence of PPP deviations among several different groups of countries. Such groups have been formed according to three different criteria; as in Curran and Velic (2019), we distinguish countries according to the industrialised-developing-emerging classification. This groups' composition is the same as in Curran and Velic (2019). We also divide countries according to their income level, following the World Bank 2010 classification. Finally, we will try to infer if any significant difference in PPP deviation exists among different geographical regions. In order to perform PCA to obtain the *core* price index we used thirteen CPI series referring to different categories of goods and services. Such series were taken from the IMF CPI database, and as for the other series they are composed of monthly data. The period covered by such series goes from 1998 to 2018. Due to scarce data availability, such series were retrieved for a subsample of twelve countries, plus the United States. The US were treated differently from the other countries, as its estimated price index was the result of a PCA applied on twenty CPI series; these data were retrieved from the FRED database, belonging to the Federal Reserve Bank of St. Louis. Details on data collected for PCA analysis can be found in the appendix.

4.2 Descriptive statistics

In the tables below we present some descriptive statistics on both the BRER and REER series. In particular, we show the mean values of both rates for each of the three country classification schemes, as well as their standard deviations. It is interesting to notice that, as we move from industrialised or high-income countries to the Developing or Low-income ones, the mean values of the BRERs tend to increase, reflecting more difficult convergence in the case of the latter ones. Regarding geographic difference, in the case of BRER we notice a significantly lower mean value for European countries' BRER, which is expected, considering that most of European countries belong to the Industrialised or High-income group. Regarding REERs, a similar pattern can be observed, but with less significant differences. Being Low-income countries REER data absent, we could not report statistics on them. It is important to notice that REER data are reported in absolute value, and that their equilibrium value corresponds to one hundred. The statistics reported are scaled so that the equilibrium value equals one. Regarding rates' variability, there is no clear pattern in both series. The only worth-mentioning result is the high volatility in Asian REERs, compared to the one of other geographical areas.

	BRER mean values	BRER standard deviations
Industrialised countries	1.54	1.14
Emerging countries	3.39	1.39
Developing countries	4.57	0.74
High-income countries	1.84	1.34
Middle-income countries	4.18	0.97
Low-income countries	5.03	0.06
Asia	4.41	0.96
Africa	3.99	0.24
Europe	1.77	1.41
America	3.66	1.22

Bilateral exchange rates descriptive statistics

	REER mean values	REER standard deviations
Industrialised countries	1.02	0.05
Emerging countries	1.03	0.15
Developing countries	1.45	1.88
High-income countries	1.02	0.10
Middle-income countries	1.28	1.48
Low-income countries	n.a.	n.a.
Asia	1.62	2.36
Africa	1.21	0.29
Europe	0.99	0.06
America	1.03	0.28

Effective exchange rates descriptive statistics

5 Empirical Results

5.1 Linear models

We are now finally able to have a look at the results we got from the empirical analysis of both bilateral and effective real exchange rates. Our aim is to compare how persistent are the deviations from Purchasing Power Parity for different categories of countries. In the case of BRERs, we look at PPP relatively to the United States, which we took as the reference country; instead, with REER we observe whether the price level of a given country is in equilibrium with the rest of the world. To begin with, we show results for the linear models described above. The figures in the table below represent the median half-life for the specific category of countries (whose exact composition is illustrated in the appendix) and for a specific model. For each of the two models, we show both the half-life (in years) computed with the formula (3) and the one obtained through the Impulse response function. Following Kilian and Zha (2002), we allowed at most a time span of forty years for a unit shock to dissipate by one-half. Countries in which the impulse response function did not fall below 0.5 within the 40-year period have been considered to be constantly far from PPP and therefore were not included in the median computation. Before analysing the results, we must underline that, in the case of BRERs, the ρ coefficient which is used for half-life computation was not significant at 10% significance level for Hong Kong, Slovenia and Syria; for REERs instead, we have non significant ρ for Portugal, China, Colombia, Russia, Saudi Arabia, Croatia, Czechia, Pakistan, Algeria, Cameroon, Morocco and Paraguay.

From a general overview of the half-lives, we immediately notice that in general we have less persistence when using the augmented model. This is likely to be due, among other reasons, to the presence of a linear trend in such model, plus the fact that the number of lags is optimally selected. Regarding BRER estimates of half-lives, we notice that for the $AR(1)$ model they fail to stay in the 3-to-5 years range which has been typically found in the literature (Kilian and Zha (2002), Cheung and Lai (2000)), while estimates coming from the augmented model are mostly within this range. REER estimates, despite being generally lower than the BRER ones, follow a similar pattern. We observe quite low half-lives values for REER estimates obtained through the augmented model. Not surprisingly, there are no relevant differences in the half-lives estimated between the two computation methods; switching from using the formula in (3) to the IRF definition does not give particular insights.

At the bottom of each categorization, we show the p-value for the Pearson χ^2 test to check whether there is a significant difference in the median of each subgroup. Analysing the table with bilateral rates' half-lives estimates, we observe that the null hypothesis of equal median half-life is rejected only in the case of the classification according to the levels of economic development, and only for the $AR(1)$ model estimates. Other p-values are all above 5%, which implies absence of statistically significant differences in median half-lives among different subgroups of countries. Focusing on the area with significant discrepancies, we see that industrialized countries take much more time than Emerging or developing ones to converge towards PPP. Although not statistically significant, we observe the exact same pattern in the level-of-income classification, with low-income countries showing much faster convergence than middle and especially high-income ones. As regards augmented model estimates, such differences are not clear enough to make a good comparison. By looking at geographical differences, for the $AR(1)$ model, we see that Europe is the region with slowest convergence, while American countries tend do be quicker. It is important to recall that in this region we include mostly Hispanic American or

Latin American countries plus Canada. Therefore, as it is pointed out in Curran and Velic (2019), it is likely that such relatively lower median half-life is due to the shorter distance from the US, which is the reference country in the BRER analysis. We could also quickly compare these results with the descriptive statistics we got: it is evident that country groups with higher mean BRER tend to have also higher median half-lives. The same pattern is not followed in the augmented model estimates, where Africa instead tends to be the slowest region to converge. Asia on the other hand, seems to be the fastest one.

Looking at the analysis of REERs, we notice that all but one p-values are higher than 5%, thus indicating lack of significant median half-life differences in each subgroup. We recall that in the REER sample there is no low-income country, that is the reason why no half-life is reported in the table. One interesting thing to notice is that, in the geographical categorisation, the trend observed in augmented model BRER estimates is replicated, with African countries showing the highest median half-life, while Asia showing the smallest. Moreover, for the half-life computed through the augmented model impulse response function, such trend is statistically significant at 5%.

5.2 ESTAR model

Below we report estimated half-lives obtained through the formula in (9), which are computed starting from the parameters of estimated ESTAR models. We report half-lives for countries with significant models, together with half-lives obtained from both $AR(1)$ and augmented AR models. As explained above, ESTAR half-lives are computed both at the mean value and at the maximum of the transition function (of which we will show some selected plots in the appendix). Where the symbol ∞

appears, we can say that according to that specific model, that country does not reach convergence in PPP within a limited period of time (above we took forty years as a reference threshold, below which the results can be considered to be economically relevant; in the next piece of analysis we will use some tolerance over such period). The most relevant aspect to notice in general is that there are clear differences between the two groups of estimates. In particular, half lives obtained through ESTAR model estimation appear to be lower than the ones coming from linear models; it is rare to observe higher half-lives among ESTAR estimates compared to the linear model ones. This is in accordance with the findings of Curran and Velic (2019) (even though they got much lower half lives even in the linear case, compared to our results). The underlying message is that, once the nonlinear feature of BRER series is considered, purchasing power parity appears to be much more concrete than what it was expected from linear models' results. Countries like France, Germany, Finland, Turkey, Ghana and El Salvador itself, despite having different levels of economic development/income and despite belonging to different geographical areas, do experience a sharp reduction in their speed of convergence to PPP, which goes from one or two decades to few years or less than one year. To close this parenthesis on ESTAR models, we might stress the fact that being estimates of nonlinear models like ESTAR very sensible to the precision of parameters' starting values, a more refined analysis is worth to be conducted in future research to obtain robust estimates for a larger pool of countries.

5.3 PCA empirical results

In this section we are going to discuss the empirical estimates of half-lives calculated using the estimated price index we obtained through the application of principal component analysis over several price index series covering thirteen categories of goods and services. As announced in the data description section, these estimates are available for a subset of countries. In table 4 we show the calculated half-life with the adjusted price index obtained from the common factor (labelled "PCA") and the ones based on standard CPI (labelled "CPI", which were also included in the median calculation in tables 1,3). This part of the analysis has been conducted using BRERs and fitting both the linear and nonlinear models explained above. Since the sample period has been shortened, we re-calculated period half-lives for all the countries in the analysis.

From a first glance, the difference between the two types of estimations seems to be impressive: focusing for a moment on the $AR(1)$ model results, we see that in the columns with CPI-based estimates most countries appear to follow a unit root process or an autoregressive one with a persistence coefficient close enough to unity to yield a half-life which goes far beyond the forty-year span. On the other hand, using the price index obtained through PCA we observe a reduction on average of PPP deviations' persistence, even though it remains substantially high for most countries (recall that we made an exception in this case regarding the forty-year threshold: indeed we show results of half-lives not longer than sixty years). There are some exceptions, like Denmark, Sweden or Norway, where half-lives deriving from the $AR(1)$ model are lower in the standard CPI version; this may be attributed to the fact that these countries, belonging all to the Scandinavian region, tend to experience similar dynamics in terms of prices of goods and services. Very similar results are obtained using the two different half-life calculation methods (formula and impulse responses).

Turning to the augmented model estimates, here we observe once again a decline in half-lives estimates in most of the analysed countries. We see that half-lives computed with standard CPI tend to lie within the 3-to-5 years range with few exceptions, but then they tend decline when the PCA estimate of the price index is introduced in place of the standard CPI; in particular, half-lives appear to be mostly in the 0-2 years range, which implies much faster convergence.

As a third instance, we observe what happens when the nonlinear model is introduced: half-lives estimates, although carried out for a relatively shorter period with respect to the one of the main analysis, tend to be shorter overall. This was indeed expected, as it reflects our findings in the above section within the linear vs. nonlinear models debate. But the aspect on which we must focus our attention is the overall reduction in ESTAR half-lives computed using the estimated price index with respect to the standard CPI ones: even though the half-lives obtained in the 'classical' way are already quite short, using the newly estimated BRERs through factor analysis we manage to obtain even shorter half-lives for most of the countries in the subsample. Exceptions are made by the Netherlands, Denmark, Norway and Sweden, which again display shorter half-lives in the standard CPI estimates. Apart from the Netherlands case, here we observe exactly the same pattern we have seen in the linear models.

Such evidence is not necessarily relevant *per se*, but it becomes of paramount importance if we connect this fact to the initial question: does an estimated version of the price index alter the behavior of a bilateral real exchange rate?[7] The answer is likely to be positive, since such discrepancy has proven to be robust to the model specification employed in the half-lives calculation.

Here we may draw a similar conclusion to the one we got from the linear-nonlinear comparison: real exchange rates are a delicate tool to be analysed, and they are

[7] And its persistence, as a consequence.

quite sensitive to the different data that can be used to construct them and to the characteristics of the models that are used to optimally capture their behaviour. Therefore, in order to study them and to carry out empirical analysis it is necessary to consider multiple approaches that can be helpful to spot and avoid potential biases or systematic irregularities.

6 Conclusions

In this thesis we analysed through the calculation of half-lives the persistence of purchasing power parity deviations in a series of countries. We started from a global analysis in which we were able to subdivide countries in different categories in order to see whether group differences in median half-lives were present. Such analysis had a mostly negative outcome, being group differences insignificant for almost all categorizations and for different half-life calculation methods and models. Secondly, we compared the fitting of linear and nonlinear models over a subset of countries, finding in most of them substantial reduction in half-lives when the nonlinear specification (the ESTAR model in our case) is employed. Lastly, we were able to show that using an estimated price index found through principal component analysis, it is possible to construct bilateral real exchange rates with less persistent deviations (shorter half-lives) than the standard CPI-based real exchange rates. Such finding is proven to be robust to both linear and nonlinear model specifications for the majority of the analysed countries.

Bilateral rates	AR(1)	AR(1)-IRF	Augmented AR	Augmented AR-IRF
Industrialised countries	14.53	14.58	3.75	3.88
Emerging countries	7.29	7.33	3.76	3.84
Developing countries	5.33	5.38	3.54	3.58
Test p-value	0.03	0.01	0.56	0.78
High-income countries	12.08	12.08	3.88	4.08
Middle-income countries	6.78	6.83	3.60	3.67
Low-income countries	4.58	4.58	2.94	3.00
Test p-value	0.20	0.26	0.54	0.56
Asia	7.79	7.83	3.22	3.25
Africa	8.40	8.41	4.99	5.00
Europe	13.32	13.33	3.41	3.42
America	5.21	5.25	3.68	3.71
Test p-value	0.26	0.30	0.56	0.49

Half lives reported in years; the p-value corresponds to the Pearson χ^2 test on group median differences. A rejection of the null implies the presence of significant median half-life differences.

Table 1: Median Half-lives with linear models: BRERs

Effective rates	AR(1)	AR(1)-IRF	Augmented AR	Augmented AR-IRF
Industrialised countries	5.08	5.12	1.91	1.96
Emerging countries	6.3	6.33	2.62	2.75
Developing countries	2.75	2.92	1.75	1.08
Test p-value	0.26	0.26	0.7	0.47
High-income countries	5.21	5.25	1.91	2
Middle-income countries	4.27	4.33	2.25	2.08
Low-income countries	n.a.	n.a.	n.a.	n.a.
Test p-value	0.34	0.32	0.75	1
Asia	4.31	4.33	2.4	1
Africa	7.09	7.17	3.75	4
Europe	4.27	4.33	1.52	1.8
America	5.13	5.25	2.45	2.5
Test p-value	0.27	0.13	0.47	0.04

Half lives reported in years; the p-value corresponds to the Pearson χ^2 test on group median differences. A rejection of the null implies the presence of significant median half-life differences.

Table 2: Median Half-lives with linear models: REERs

Bilateral rates	AR(1)	Augmented AR-IRF	ESTAR_mean	ESTAR_max
Canada	7.98	5.50	0.76	0.60
France	15.42	4.08	0.05	0.05
Germany	11.02	3.25	0.24	0.21
Italy	17.09	5.25	0.71	0.64
Portugal	18.75	5.00	2.19	1.25
Spain	18.57	5.75	10.62	5.55
Netherlands	15.24	3.67	1.92	1.31
Switzerland	12.08	2.75	1.28	0.86
United Kingdom	3.84	2.58	0.18	0.17
Belgium	17.27	7.17	0.05	0.05
Denmark	6.18	3.42	0.36	0.39
Finland	14.55	3.67	1.15	0.98
Luxembourg	17.24	13.17	3.34	1.96
Norway	5.81	3.17	0.56	0.64
Sweden	9.04	3.33	0.73	0.78
Singapore	7.29	5.17	1.19	1.08
Slovenia	∞	8.67	21.50	9.14
Turkey	4.43	5.00	0.24	0.24
Ghana	8.80	3.75	1.10	0.49
El Salvador	26.27	3.67	5.84	3.63

Half-lives reported in years. "mean" and "max" respectively stand for the evaluation of the half-life at the mean and at the 100^{th} percentile of the transition function. The ∞ symbol indicates a half-life > 40 years

Table 3: Half-lives significant estimates with ESTAR model

	AR(1)		AR(1)-IRF		Augmented AR-IRF		$ESTAR_{mean}$		$ESTAR_{max}$	
Bilateral Rates	CPI	PCA	CPI	PCA	CPI	PCA	CPI	PCA	CPI	PCA
France	∞	18.35	∞	18.42	2.83	0.92	3.21	0.27	0.60	0.26
Germany	∞	22.79	∞	22.83	4.25	2.92	1.99	0.17	0.47	0.15
Italy	∞	56.46	∞	∞	∞	1.00	2.92	1.43	0.74	0.26
Spain	∞	∞	∞	∞	3.50	6.92	2.46	1.32	0.59	0.46
Netherlands	∞	5.71	∞	5.75	5.08	1.42	2.29	12.95	0.54	5.78
United Kingdom	3.42	3.84	3.50	3.92	2.08	2.25	1.39	0.25	1.21	0.24
Denmark	3.36	41.37	3.42	∞	3.08	0.92	3.30	7.25	0.69	1.61
Luxembourg	∞	0.26	∞	0.33	3.58	0.25	2.44	1.44	0.59	0.47
Norway	3.69	5.75	3.75	5.75	3.42	5.08	0.10	0.23	0.08	0.23
Sweden	3.48	27.09	3.50	27.17	2.58	2.00	0.47	10.10	0.33	∞
Portugal	∞	3.47	∞	3.50	3.25	1.83	2.61	0.17	0.62	0.13
Finland	∞	10.24	∞	10.25	2.67	1.50	3.19	1.25	0.61	0.95

Half-lives in years; the "CPI" column refers to half-lives computed by employing the standard CPI in the BRER construction; the "PCA" column refers instead to the half-lives computed with the factor-adjusted Bilateral real exchange rate;

The ∞ symbol indicates a half-life > 60 years

Table 4: CPI - PCA index comparison for half-lives estimates with all models

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Appendix

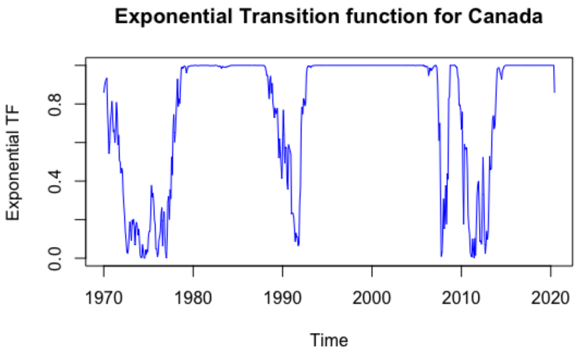
In this brief Appendix we will firstly show the plots of some ESTAR Transition Function (TF) examples. We will then see in detail the price index series employed for the Principal component analysis both for the US and the other twelve countries involved. As a third item, the actual coefficients of some selected countries for both the linear and the nonlinear model are shown. Finally, we will provide the composition of the categories in which countries have been divided.

It is interesting to observe how, over time, the Transition Function behaves differently for the analysed countries; some countries, like Canada, France and Germany for instance, experience quite sharp transitions between regimes, while for Singapore and the Netherlands we have a smoother process. It is worth noticing that France, Finland and the Netherlands all transit into the 'inner' regime in the year 2000.

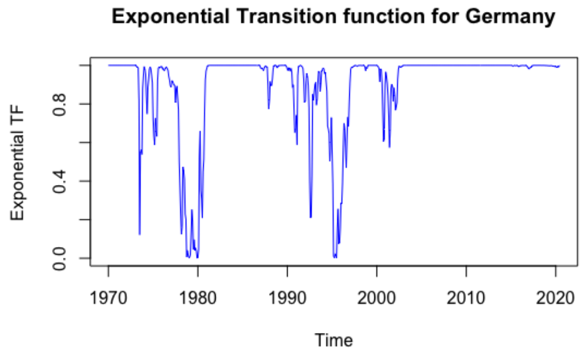
As we already knew, ESTAR coefficients for Russia, Bolivia and Uruguay are not significant, hence not useful for half-life computation. Coefficients for linear models, on the other hand, tend to be mostly significant (here we chose to show the $AR(p)$ model specification, being the one subject to lag selection).

Regarding country classification, we recall that the one which divides countries according to the level of economic development is taken from Curran and Velic (2019), while the one which follows the classification according to the level of income is taken from the World Bank classification. Looking at geographies, we decided to put together Southern, Central and Northern-American countries, calling their region simply 'Americas'.

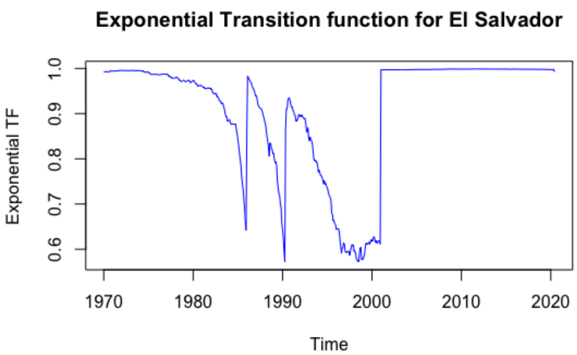
ESTAR Transition Function plots



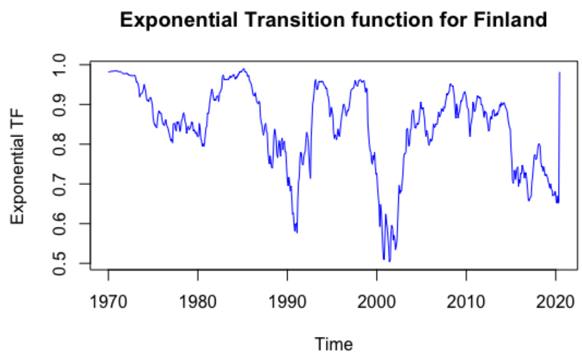
(a) Canada TF



(b) Germany TF

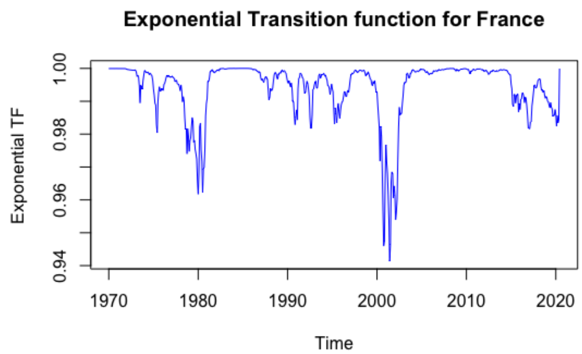


(c) El Salvador TF

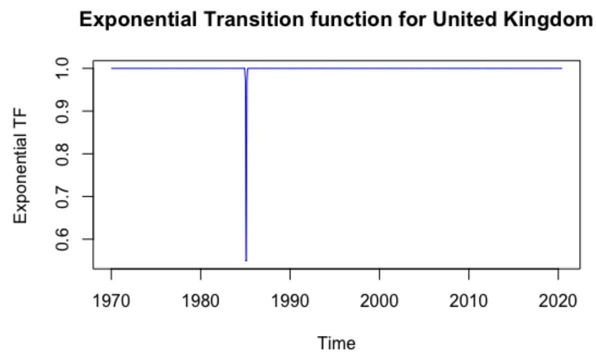


(d) Finland TF

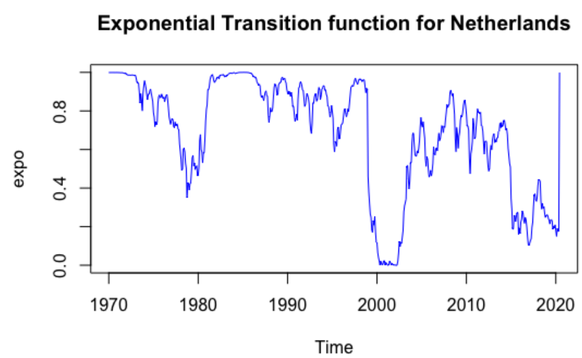
Figure 1: TF Plots



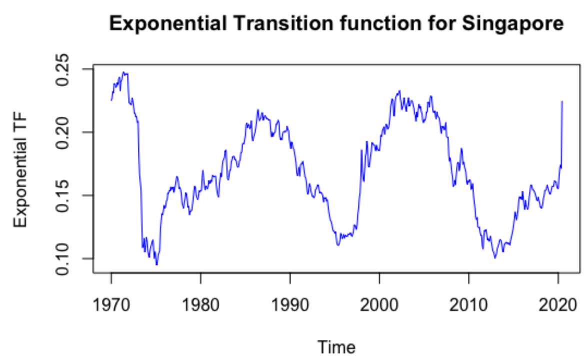
(a) France TF



(b) UK TF



(c) Netherlands TF



(d) Singapore TF

Figure 2: TF plots

Price index series

Series code	Series description
CPIAUCSL	All Items in U.S. City Average
CPILFESL	All Items Less Food and Energy in U.S. City Average
CUUR0000SEHA	Rent of Primary Residence in U.S. City Average
CPIMEDSL	Medical Care in U.S. City Average
CPIFABSL	Food and Beverages in U.S. City Average
CPIHOSNS	Housing in U.S. City Average
CPIAPPSL	Apparel in U.S. City Average
CPIUFDNS	Food in U.S. City Average
CUUR0000SETA01	New Vehicles in U.S. City Average
CPITRNSL	Transportation in U.S. City Average
CUSR0000SAM2	Medical Care Services in U.S. City Average
CPIENGSL	Energy in U.S. City Average
CUUR0000SETB01	Gasoline (All Types) in U.S. City Average
CUSR0000SEEB	Tuition, Other School Fees, and Childcare in U.S. City Average
CUSR0000SAH1	Shelter in U.S. City Average
CUSR0000SAD	Durables in U.S. City Average
CUSR0000SEHF01	Electricity in U.S. City Average
CUSR0000SAC	Commodities in U.S. City Average
CUSR0000SEMC	Professional Services in U.S. City Average
CUUR0000SEGA	Tobacco and Smoking Products in U.S. City Average

Table 5: US price index series employed in PCA. *Source: FRED database*

Series Code	**Series description**
PCPI_IX	Consumer Price Index, All items
PCPIF_IX	Food and non-alcoholic beverages
PCPIFBT_IX	Alcoholic Beverages, Tobacco, and Narcotics
PCPIA_IX	Clothing and footwear
PCPIH_IX	Housing, Water, Electricity, Gas and Other Fuels
PCPIHO_IX	Furnishings, household equipment and routine household maintenance
PCPIM_IX	Health
PCPIT_IX	Transport
PCPIEC_IX	Communication
PCPIR_IX	Recreation and culture
PCPIED_IX	Education
PCPIRE_IX	Restaurants and hotels
PCPIO_IX	Miscellaneous goods and services

Table 6: Price index series for countries involved in PCA. *Source: IMF CPI database*

Model coefficients

Italy

	Coefficient	Std. Error	t-statistic	p-value
β_0	0.1351	0.0737	1.8329	0.0673
μ	-0.0003	0.0002	-1.9052	0.0572
ρ	-0.0147	0.0071	-2.0890	0.0371
β_1	0.0037	0.0417	0.0885	0.9295

Portugal

	Coefficient	Std. Error	t-statistic	p-value
β_0	0.0895	0.0520	1.7207	0.0858
μ	-0.0002	0.0001	-1.7999	0.0724
ρ	-0.0139	0.0069	-2.0071	0.0452
β_1	0.0024	0.0416	0.0576	0.9541

Spain

	Coefficient	Std. Error	t-statistic	p-value
β_0	0.0908	0.0515	1.7641	0.0782
μ	-0.0002	0.0001	-1.8351	0.0670
ρ	-0.0148	0.0072	-2.0575	0.0401
β_1	0.0047	0.0416	0.1123	0.9107

Belgium

	Coefficient	Std. Error	t-statistic	p-value
β_0	0.0612	0.0353	1.7308	0.0840
μ	-0.0001	0.0001	-1.8021	0.0720
ρ	-0.0144	0.0071	-2.0363	0.0422
β_1	0.0064	0.0416	0.1528	0.8786

Luxembourg

	Coefficient	Std. Error	t-statistic	p-value
β_0	0.0602	0.0349	1.7268	0.0847
μ	-0.0001	0.0001	-1.7992	0.0725
ρ	-0.0141	0.0070	-2.0250	0.0433
β_1	0.0098	0.0416	0.2348	0.8145

Russia

	Coefficient	Std. Error	t-statistic	p-value
β_0	0.1430	0.0490	2.9162	0.0038
μ	-0.0000	0.0001	-0.0697	0.9445
ρ	-0.0453	0.0173	-2.6213	0.0092
β_1	-0.0678	0.0582	-1.1657	0.2447

Bolivia

	Coefficient	Std. Error	t-statistic	p-value
β_0	0.0725	0.0356	2.0330	0.0425
μ	0.0000	0.0000	0.4884	0.6255
ρ	-0.0419	0.0209	-2.0040	0.0455
β_1	-0.0868	0.0449	-1.9335	0.0537

Uruguay

	Coefficient	Std. Error	t-statistic	p-value
β_0	0.0629	0.0277	2.2727	0.0234
μ	-0.0000	0.0000	-0.8864	0.3758
ρ	-0.0187	0.0080	-2.3321	0.0200
β_1	-0.0279	0.0409	-0.6815	0.4958

Table 7: Linear AR(p) model coefficients (BRERs)

Italy				
	Coefficient	Std. Error	t-statistic	p-value
ρ	-0.0010	0.0000	-62.1625	0.0000
ρ^*	-0.0079	0.0001	-109.9506	0.0000
β_1	1.0010	0.0527	18.9805	0.0000
β_1^*	0.0067	0.0173	0.3887	0.6977
α	0.0079	9.3172	0.0008	0.9993
α^*	-0.0663	0.0008	-83.7204	0.0000
γ	0.0057	0.0020	2.8912	0.0040
th	9.1570	0.0805	113.8003	0.0000

Portugal				
	Coefficient	Std. Error	t-statistic	p-value
ρ	-0.0382	0.0224	-1.7095	0.0879
ρ^*	0.0389	0.0223	1.7426	0.0819
β_1	1.0345	0.0201	51.4261	0.0000
β_1^*	-0.0351	0.0200	-1.7528	0.0802
α	0.0090	0.0060	1.4947	0.1355
α^*	-0.0123	0.0056	-2.1775	0.0298
γ	1.9373	0.3673	5.2749	0.0000
th	1.3520	0.4195	3.2232	0.0013

Spain				
	Coefficient	Std. Error	t-statistic	p-value
ρ	-0.0248	0.0124	-2.0038	0.0455
ρ^*	0.0254	0.0123	2.0590	0.0399
β_1	1.0226	0.0113	90.6494	0.0000
β_1^*	-0.0232	0.0112	-2.0641	0.0394
α	0.0064	0.0038	1.6789	0.0937
α^*	-0.0092	0.0035	-2.6486	0.0083
γ	1.9112	0.3354	5.6987	0.0000
th	1.2414	0.3685	3.3688	0.0008

Belgium				
	Coefficient	Std. Error	t-statistic	p-value
ρ	-0.0208	0.0075	-2.7523	0.0061
ρ^*	0.0215	0.0075	2.8710	0.0042
β_1	1.0187	0.0069	147.3576	0.0000
β_1^*	-0.0194	0.0068	-2.8370	0.0047
α	0.0028	0.0015	1.9167	0.0557
α^*	-0.0052	0.0012	-4.3018	0.0000
γ	2.4760	0.3650	6.7833	0.0000
th	0.7562	0.2126	3.5562	0.0004

Table 8: ESTAR model coefficients

Luxembourg				
	Coefficient	Std. Error	t-statistic	p-value
ρ	-0.0204	0.0070	-2.8985	0.0039
ρ^*	0.0211	0.0070	3.0381	0.0025
β_1	1.0187	0.0065	156.1588	0.0000
β_1^*	-0.0194	0.0064	-3.0061	0.0028
α	0.0030	0.0015	2.0322	0.0426
α^*	-0.0055	0.0012	-4.7060	0.0000
γ	2.3971	0.3291	7.2843	0.0000
th	0.7588	0.2068	3.6693	0.0003

Russia				
	Coefficient	Std. Error	t-statistic	p-value
ρ	0.0049	0.0013	3.7221	0.0002
ρ^*	0.0562	0.6320	0.0889	0.9293
β_1	0.9951	0.0013	754.8647	0.0000
β_1^*	-0.0560	0.6298	-0.0889	0.9292
α	-0.0016	0.0004	-3.5689	0.0004
α^*	-0.5121	5.4394	-0.0941	0.9251
γ	0.0012	0.0136	0.0900	0.9284
th	-0.6519	0.2760	-2.3620	0.0188

Bolivia				
	Coefficient	Std. Error	t-statistic	p-value
ρ	-0.0018	0.0078	-0.2240	0.8229
ρ^*	-0.2357	94.2758	-0.0025	0.9980
β_1	1.0018	0.0080	125.0663	0.0000
β_1^*	0.2080	83.2364	0.0025	0.9980
α	0.0067	0.0391	0.1719	0.8636
α^*	-1.4317	565.3564	-0.0025	0.9980
γ	0.0001	0.0209	0.0026	0.9980
th	5.0198	8.2547	0.6081	0.5433

Uruguay				
	Coefficient	Std. Error	t-statistic	p-value
ρ	23.2434	91.0605	0.2553	0.7986
ρ^*	-23.2215	91.0615	-0.2550	0.7988
β_1	-22.9382	93.7612	-0.2446	0.8068
β_1^*	23.9171	93.7621	0.2551	0.7987
α	-41.4496	164.4256	-0.2521	0.8011
α^*	41.3498	164.4292	0.2515	0.8015
γ	0.0199	0.0107	1.8696	0.0620
th	-6.3696	5.1681	-1.2325	0.2183

Table 9: ESTAR model coefficients

Economic Devlopment

- **Industrialised countries:** United States, Canada, France, Germany, Italy, Portugal, Spain, Netherlands, Switzerland, United Kingdom, Belgium, Denmark, Finland, Japan, Luxembourg, Norway, Sweden.

- **Emerging countries:** Brazil, China, Colombia, India, South Korea, Mexico, Russia, Saudi Arabia, South Africa, Chile, Croatia, Czechia, Ecuador, Hong Kong SAR China, Indonesia, Pakistan, Peru, Philippines, Singapore, Slovenia, Thailand, Turkey.

- **Developing countries:** Bolivia, Botswana, Ghana, Côte d'Ivoire, Iran, Jamaica, Nigeria, Syria, Algeria, Cameroon, Congo - Kinshasa, Costa Rica, El Salvador, Haiti, Kenya Morocco, Paraguay, Rwanda, Senegal, Sri Lanka, Uruguay, Ukraine.

Level of income (WB classification)

- **High-income countries:** United States, Canada, France, Germany, Italy, Portugal, Spain, Netherlands, Switzerland, United Kingdom, Belgium, Denmark, Finland, Japan, Luxembourg, Norway, Sweden, South Korea, Saudi Arabia, Croatia, Czechia, Hong Kong SAR China, Singapore, Slovenia.

- **Middle-income countries:** Brazil, China, Colombia, India, Mexico, Russia, South Africa, Chile, Ecuador, Indonesia, Pakistan, Peru, Philippines, Thailand, Turkey, Bolivia, Botswana, Ghana, Côte d'Ivoire, Iran, Jamaica, Nigeria, Syria, Algeria, Cameroon, Congo - Kinshasa, Costa Rica, El Salvador, Morocco, Paraguay, Senegal, Sri Lanka, Uruguay, Ukraine.

- **Low-income countries:** Rwanda, Haiti, Kenya.

Geographical areas

- **Asia:** China, Hong Kong SAR China, India, Indonesia, Iran, Japan, South Korea, Pakistan, Philippines, Saudi Arabia, Singapore, Syria, Thailand, Turkey, Sri Lanka.
- **Africa:** Algeria, Botswana, Cameroon, Congo - Kinshasa, Côte d'Ivoire, Ghana, Kenya, Morocco, Nigeria, Rwanda, Senegal, South Africa.
- **Europe:** Belgium, Croatia, Czechia, Denmark, Finland, France, Germany, Italy, Luxembourg, Netherlands, Norway, Portugal, Russia, Slovenia, Spain, Switzerland, Sweden, United Kingdom, Ukraine.
- **Americas:** United States, Bolivia, Brazil, Chile, Colombia, Costa Rica, Ecuador, El Salvador, Haiti, Jamaica, Mexico, Paraguay, Peru, Uruguay, Canada.

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