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Heterogeneity in the Response of the Euro Area Economies to Unexpected Monetary Policies of the ECB

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Abstract

The objective of this study is to investigate potential disparities among Euro area member states in their responses to unforeseen monetary policy measures implemented by the European Central Bank (ECB). This analysis entails the estimation of impulse response functions for national variables, which illustrate their reactions to innovations in the monetary policy instrument. To characterize the behavior of these variables and derive the impulse responses, I employ the Vector Error Correction Model (VECM) parameterization.

This research centers on national economies, which are represented by their industrial production and unemployment rates. Notably, both these variables are integrated across all economies, just as the monetary policy instruments are. It is essential to employ these series in their original levels, as doing so allows to capture long-term relationships among the variables. While Vector Autoregression (VAR) models are typically utilized to describe the behavior of stationary and non-integrated series and capture short-term relationships, the VECM accommodates integrated variables, even potentially cointegrated ones, thus enabling to grasp both short-term and long-term relations. Moreover, this parameterization proves valuable for modeling the unique attributes of the Euro area, which the Optimal Currency Area (OCA) theory identifies in the facilitation of imports, exports, and labor mobility among member states. Additionally, it accounts for the shared currency among diverse economies that are influenced by the same monetary policy. This structural framework further allows to incorporate the new Keynesian concept of equilibrium in unemployment, which serves as the channel through which monetary policy affects the real economy.

The analysis involves estimating the parameters of three distinct models, each focusing on a specific monetary policy instrument. These models encompass both conventional and unconventional monetary policy instruments employed by the ECB. Subsequently, impulse response functions are derived from the three estimated VECM models, one for each monetary policy variable adopted. The resulting functions provide evidence of heterogeneity in the responses of Euro area economies to both conventional and unconventional policies, particularly concerning production. Notably, France, Spain, and Italy do not exhibit increased production in response to the ECB's expansionary monetary policies. This suggests that the ECB's attempts to stimulate these economies through monetary policy actions may yield less effective results, potentially leaving specific member states at a disadvantage when such actions are implemented. Moreover, this diversity in responses contributes to an increase in the disparities among the monetary union's economies, consequently resulting in higher macroeconomic costs associated with being part of this union.

Introduction

Investigating how various European national economies respond to the actions of the European Central Bank (ECB) is of paramount importance due to the structural significance and global influence of the Euro area. In cases where heterogeneity exists in the responses of Euro area member states, the ECB might adopt suboptimal monetary policy measures for the Euro area economies, potentially impacting the global economy. De Grauwe and Mongelli (2005) argued that the Euro area economies were not yet sufficiently integrated to generate efficiency gains that could offset the macroeconomic costs of the union. According to the Optimal Currency Area (OCA) theory, particularly as outlined by Frankel and Rose (1998), the advantages of joining a currency area stem from reduced transaction costs and increased trade in goods and services among member economies. Conversely, the costs of joining a monetary union arise from the inability of national economies to implement monetary policies tailored to their unique characteristics. The optimality of a currency area is, therefore, linked to the reduction of "distances" in terms of macroeconomic and structural characteristics among member states, resulting in greater homogeneity and lower costs associated with union membership. Consequently, it is particularly intriguing to discern whether such heterogeneity exists among Euro area member states in their responses to unexpected monetary policy measures implemented by the ECB. This analysis seeks to achieve this objective by characterizing the diverse national responses encountered by the ECB when it unexpectedly implements its monetary policies.

According to the new Keynesian theory of money, monetary policies impact real economic variables by influencing the equilibrium in employment, primarily due to the stickiness of prices and wages. When modeling the behavior of these variables within the Euro area national economies, it is crucial to consider specific key characteristics associated with monetary unions, as elucidated by the OCA theory. Mundell (1961), Frankel and Rose (1998), Goodhart(1998), De Grauwe and Mongelli (2005), among others, emphasize the significance of the facilitation of imports, exports, and labor mobility among member states of a currency union, as well as the presence of a common currency leading to a unified monetary policy which influences all member state economies. In this context, national economies are represented by their respective national unemployment rates and their industrial production, which serves as a proxy for their economic performance. The analysis entails the collection of monthly data spanning from January 1999 to March 2023, encompassing a period during which the ECB had the capacity to influence Euro area member states through actions taken on the Euro.

Monetary policy effects have been extensively analyzed in recent years. The literature predominantly employs

Vector Autoregression (VAR) models to describe the processes of variables and subsequently derive the impact of monetary policies. Some studies, such as those by Christiano, Eichenbaum, and Evans (1998), Ljungqvis and Uhlig (1999), and Peersman (2011), have utilized VAR models. Additionally, Boeckx et al. (2014) involved the use of Structural VAR (SVAR) models to analyze the impact of balance sheet interventions on the European economy, revealing that Euro area output and consumer prices rise after an increase in the ECB's balance sheet. Gertler and Karadi (2015) employed a SVAR model to estimate the impact of a monetary policy tightening by the Fed on US output, finding a significant reduction in output. Moreover, Burriel and Galesi (2018) implemented a SVAR model that explicitly considered national macro-financial dynamics and cross-country interdependencies. This approach allowed them to better capture the relevant transmission channels of monetary policy and assess the size and dispersion of the effects of unconventional monetary policy shocks across Euro area countries. They found that unconventional monetary policies had beneficial effects on aggregate output and inflation, confirming their role as a stabilization tool. However, at the disaggregated level, they observed that national economies benefited differently from unconventional monetary policies, indicating a substantial degree of heterogeneity.

The Vector Error Correction Models (VECM) have also been used in the literature to analyze monetary policies. For instance, Sun, Gan, and Hu (2010) employed the VECM to uncover the long-run relationships connecting monetary policy indicators, bank balance sheet variables, and macroeconomic variables in China. Asari et al. (2011) obtained impulse response functions from a VECM model that incorporates variables such as interest rates, inflation rates, and exchange rates. Their objective was to comprehend the effects of shocks on these variables, and they concluded that, by considering long-term relationships, interest rate policies do effectively mitigate exchange rate volatility. De Mello and Pisu (2010) conducted a test to assess the presence of a lending channel in the transmission of monetary policy within the Brazilian context using a VECM framework. Their findings suggest that monetary policy contributes to reestablishing equilibrium in the credit market by influencing the interest rate at which banks can obtain non-deposit funds. Additionally, Agbonlahor (2014) adopted the VECM parameterization to investigate the impact of monetary policies on economic growth in the United Kingdom, revealing that money supply is a significant monetary policy instrument driving growth. I selected the VECM parameterization for this analysis due to its ability to incorporate the above mentioned key features of a currency union and its capacity to work with cointegrated variables. This capability is essential for capturing long-term relationships among the variables. In contrast, adopting a VAR parameterization in this context would only be feasible if the collected data (all integrated) were transformed into their rate of change, rendering them stationary. However, such a transformation would result in the variables losing their long-term relationships, providing impulse responses that reflect only the short-term impact of an innovation in the monetary policy instrument. Consequently, an analysis conducted through a VAR model would miss possibly two critical pieces of information: the potential cointegration of the variables, rather than simple integration, and their long-term relationships.

I estimated three distinct VECM models, each corresponding to a specific monetary policy instrument, which includes one conventional and two unconventional instruments. The impulse responses of national economies to an unforeseen monetary policy action (both conventional and unconventional) by the ECB, derived from the estimated VECM models, provide evidence of heterogeneity in the responses of Euro area economies, particularly in terms of production. In particular, France, Spain, and Italy do not exhibit increased production in response to the ECB's expansionary monetary policies when considering the employment transmission channel. This suggests that the ECB's attempts to stimulate these economies through monetary policy actions may yield less effective results, potentially leaving specific member states at a disadvantage when such actions are implemented. Moreover, this diversity in responses contributes to an increase in the disparities among the monetary union's economies, consequently resulting in higher macroeconomic costs associated with being part of this union. Notably, the results derived from considering unconventional monetary policies align with the findings of Burriel and Galesi (2018), both in terms of the beneficial effects on output, and the heterogeneity among Euro area member states in their responses.

This analysis proceeds as follows: Chapter 1 presents the data collected for the national economies, as well as the monetary policy variables adopted, and further explains how these data are manipulated for estimating the model's parameters. Chapter 2 introduces the VECM model adopted together with its key characteristics, and outlines the derivation of the impulse responses, including their asymptotic distribution. Finally, Chapter 3 presents the analysis results for three different measures of monetary policy: interest rates (a conventional monetary policy instrument), the stock of M2 (an unconventional monetary policy instrument), and the stock of M3 (another unconventional monetary policy instrument).

Data

Section 1.1 introduces the variables representing the national economies, specifically industrial production and the unemployment rate, along with the conventional and unconventional monetary policy instruments considered in this analysis. In a VECM parameterization, each endogenous variable enters lagged as an exogenous variable in all equations of the system. Therefore, considering at least two variables per member state, plus the monetary policy instrument, would result in a very large number of parameters to be estimated. The challenge arises when working with a limited sample size, as estimating numerous parameters can reduce the model's statistical power and hinder its ability to make accurate inferences. Consequently, Section 1.2 shows how the variables of interest are modeled to reduce the model's dimensions and the number of parameters to be estimated, thereby ensuring the reliability of the impulse response functions derived from the model.

1.1 National Variables and Monetary Policy Instruments

Starting from January 1^{st} 1999 the Euro became scriptural money under the control of the ECB, although it did not circulate until January 1^{st} 2002, and did not become the unique currency of the Euro area until March 1^{st} 2002. The objective of this analysis is to comprehend the diverse effects of an unexpected monetary policy enacted by the ECB on the national economies of the Euro area. To achieve this, I have selected a data-set covering the period from January 1999 to March 2023, with a monthly frequency. This time-frame was chosen because it encompasses the period during which the ECB had the capability to directly influence the economies of the Euro area through its monetary policy actions, making it a relevant window for examining the impact of these actions on national economies, which is the primary focus of this analysis.

In January 1999, the Euro area comprised eleven states, namely: Austria, Belgium, Finland, France, Germany, Ireland, Italy, Luxembourg, Netherlands, Portugal, and Spain. These economies are represented in the model by the natural logarithm of their national Industrial Production (IP) and their national Unemployment Rate (UR). Although Gross Domestic Product (GDP) would be a better index of the states' economic performance, monthly data for this variable are not available. The choice of using industrial production as proxy for GDP is thus due to the availability of data at monthly frequency, and is extensively adopted with the same purpose in the existing literature.

The historical data for the aforementioned national variables were sourced from the Organization for Economic Cooperation and Development (OECD). Industrial production, as defined by the OECD, represents the output of industrial establishments and encompasses sectors such as mining, manufacturing, electricity, gas, steam, and air-conditioning. This variable serves as an indicator of the national economy's overall performance and is influenced by both the availability of capital and the size of the labor force engaged in production. As a result, both of these factors are incorporated into the model. The unemployment rate, another key indicator, is measured as the number of unemployed individuals expressed as a percentage of the total labor force. The OECD's definition of unemployed individuals includes those of working age who are currently without employment, actively seeking work, and available for work. The labor force is defined as the sum of individuals who are currently employed and those who are unemployed according to the provided criteria. These two definitions for both variables are applied uniformly across all member states by the OECD, thus ensuring an accurate comparability of the data across economies.

Central banks historically exert influence on economies by manipulating interest rates. The interest rate can be viewed as the cost associated with borrowing or lending capital and, therefore, functions as a price. In the context of the Euro area, firms do not have direct access to borrowing or lending with the ECB. Consequently, the transmission of monetary policy to firms occurs indirectly through banks as they adjust their credit standards. The ECB's direct alteration of the interest rate is referred to as "conventional monetary policy." However, since 2014, the ECB has implemented what are known as "unconventional monetary policies." These policies encompass a range of measures designed to stimulate the economy and manage inflation by influencing the money supply. While these unconventional policies indirectly impact interest rates, their primary objective is to increase or decrease the overall liquidity within the economy. For this reason, this kind of policies are mostly adopted to provide stability rather than with the objective of stimulating the economy. To comprehensively analyze both conventional and unconventional monetary policy instruments, this study considers three distinct measures of monetary policy: interest rates, the stock of the monetary aggregate M2[1], and the stock of the monetary aggregate M3[2]. Time series data for these three monetary policy measures are obtained from the ECB's Statistical Data Warehouse. As a proxy for interest rates, the minimum bid rate on the ECB's Main Refinancing Operations (MRO) is employed, following the approach in Peersman (2011). For M2, the variable is collected as a level relative to a base year and is considered in its natural logarithm. Finally, M3 is collected in millions of euros and, similar to M2, is analyzed in its natural logarithm.

Figure 1.1a presents the data on industrial production for the core (red) and excluded (blue) economies, while Figure 1.1b displays the industrial production time series of the intermediate (red) and peripheral (blue) economies. These clusters have been estimated by Campos and Macchiarelli (2021). Further information about these subgroups can be found in Section 1.2, where the data manipulation process aimed at reducing the dimensionality of the VECM model to be estimated is thoroughly explained.

Figure 1.2 presents the unemployment rate data for national economies. The clustering and the presentation

[1] M2 is defined by the ECB as the sum of currency in circulation, overnight deposits, deposits with an agreed maturity of up to two years, and deposits redeemable at notice of up to three months.

[2] M3, as defined by the ECB, represents the sum of M2, repurchase agreements, money market fund shares/units, and debt securities with a maturity of up to two years.

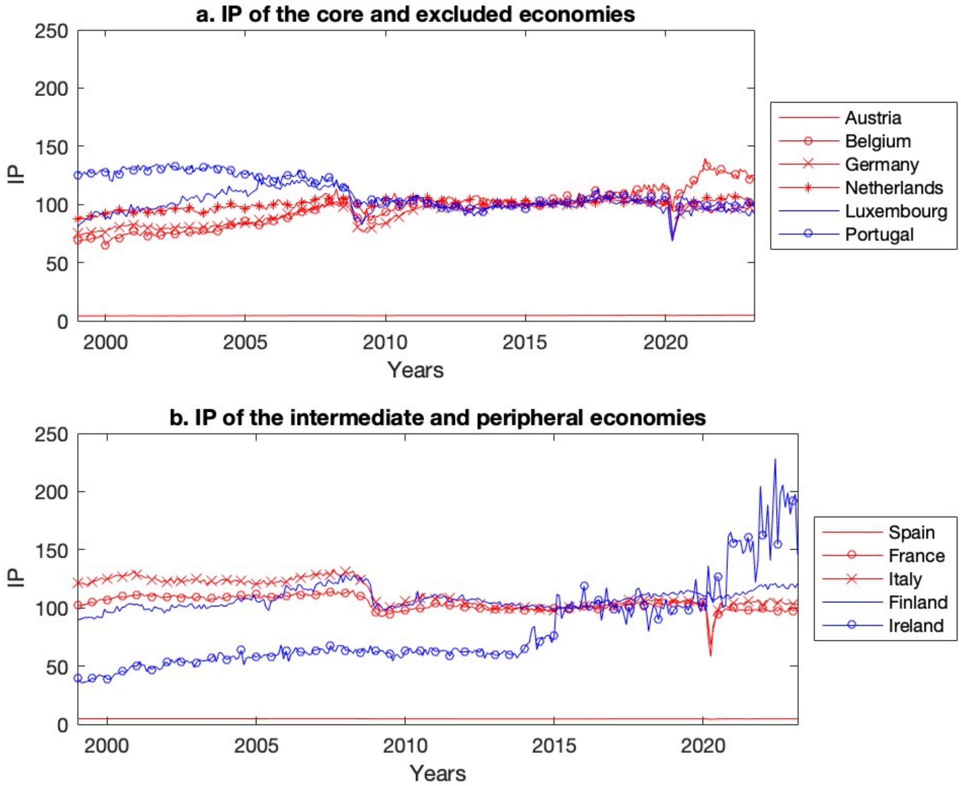

Figure 1.1: plot (from January 1999 to March 2023) of industrial production (IP) for the eleven countries which compose the Euro area since January 1999 (Austria, Belgium, Finland, France, Germany, Ireland, Italy, Luxembourg, Netherlands, Portugal, and Spain). IP is plotted as relative level of IP compared to the base year, which is set in 2015. The economies in panel a. compose the core (in red) and excluded (in blue) economies, while in panel b. the intermediate (in red) and peripheral (in blue) economies are found. These distinctions derive from Campos and Macchiarelli (2021) and are further analyzed in section 1.2.

of economies in this figure follow the same pattern as in Figure 1.1, which represents industrial productions of the member states. An important observation is the varying levels of volatility in unemployment rates across different states. For instance, in countries like Germany and Austria, the unemployment rate exhibits relatively stable patterns. In contrast, economies such as Spain, Ireland, and Portugal demonstrate a notably higher sensitivity of unemployment rates to macroeconomic factors. This figure, in comparison to the one depicting industrial production, provides a clearer picture of the existing heterogeneity within the structure of national economies in the Euro area. It is worth emphasizing that industrial production is intricately linked to the size of the labor force engaged in economic activities.

Figure 1.3 displays the three variables used as indicators of the ECB's monetary policy, namely the interest rate (Figure 1.3a), the monetary aggregate M2 (Figure 1.3b), and the monetary aggregate M3 (Figure 1.3c). The analysis focuses on expansionary monetary policies, which are characterized by either a reduction in the interest rate or an increase in the stock of the monetary aggregate (M2 or M3, as defined by the ECB[3]). The monetary aggregate level, set by the ECB, determines the money supply, while the interest rate serves as the price of capital, establishing market equilibrium by balancing demand and supply. Both a decrease in the interest

[3] M1 is composed of the sum of currency in circulation and overnight deposits. Although part of the monetary aggregates, it is not considered in this analysis.

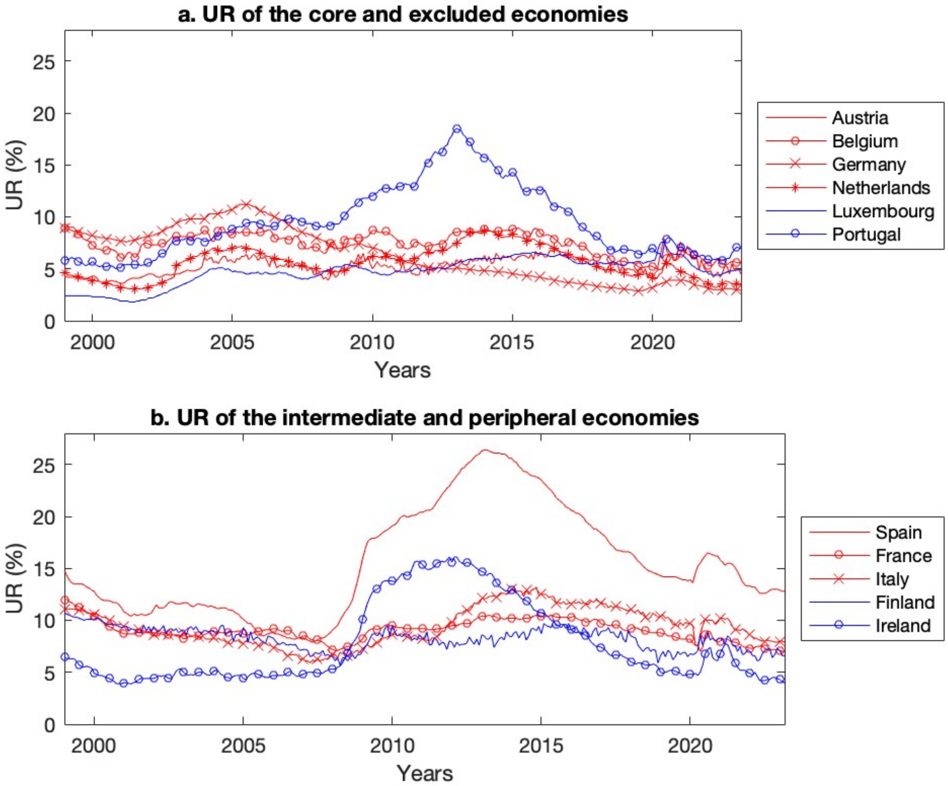

Figure 1.2: plot (from January 1999 to March 2023) of the unemployment rate (UR) for the eleven countries that compose the Euro area since January 1999 (Austria, Belgium, Finland, France, Germany, Ireland, Italy, Luxembourg, Netherlands, Portugal, and Spain). UR is plotted in percent (%). The economies in panel a. compose the core (in red) and excluded (in blue) economies, while in panel b. the intermediate (in red) and peripheral (in blue) economies are found. These distinctions derive from Campos and Macchiarelli (2021) and are further analyzed in section 1.2.

rate and an increase in the money supply are considered expansionary monetary policies because an augmented money supply leads to lower prices of capital, thus of the interest rates. This policy is labeled as "expansionary" since it is expected to have a positive impact on the economic performance of national economies. As the cost of capital decreases, it becomes more affordable for firms to obtain capital, which they can then utilize for investments and hiring workers. Consequently, after the implementation of an expansionary monetary policy, the theory predicts an increase in the industrial production variable and a decrease in the unemployment rate variable (indicating higher levels of employment in the labor force). Results in line with this predictions are found in the context of the Euro area by Peersman (2011) and Burriel and Galesi (2018).

1.2 Principal Component Analysis

One of the objectives of the model is to allow the national economies to interact with each other, which is a key feature of the currency unions, as emphasized by Mundell (1961), Frankel and Rose (1998), Goodhart(1998), De Grauwe and Mongelli (2005). Imports and exports are facilitated across the currency area, as well as it is for labour mobility. Including all of the states in both the variables representing them, plus the monetary policy variable, would make a total of twenty-three endogenous variables and possibly many more parameters to be estimated. This since in a VECM parameterization each endogenous variable enters lagged as an exogenous

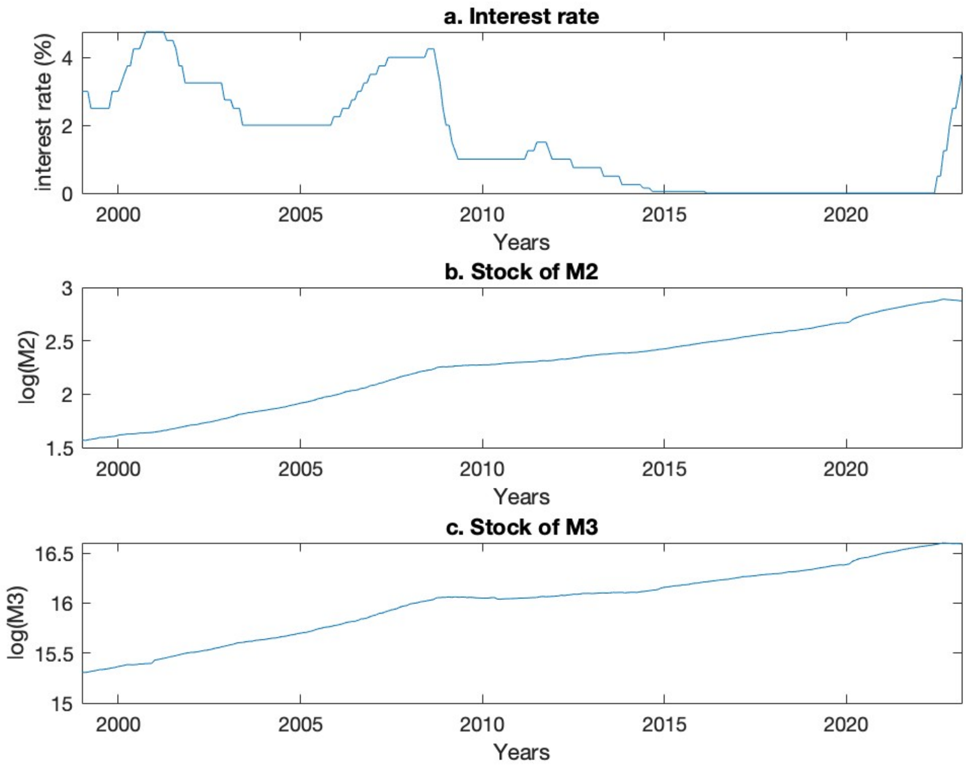

Figure 1.3: plot (from January 1999 to March 2023) of the three monetary policy variables: in panel a. the interest rates (%), in panel b. the natural logarithm of the stock of M2 (normalized with respect to a base year which sets the 100 level), and in panel c. the natural logarithm of the stock of M3 (in millions of euros).

variable in all equations of the system. This would pose problems in the estimation and inference of the model, since the sample size only comprises monthly data from January 1999 to March 2023, for a total of 291 observations. With a limited sample size, the statistical power of such a model would be very low, and the inference would potentially have no reliability. For this reason, the national economies in this analysis are clustered in three different groups, with the objective of reducing the number of parameters to be estimated, while still capturing the characteristics of the member states being connected in terms of trades and labor mobility, as extensively demonstrated by the OCA theory. The national economies are assigned to a group following Campos and Macchiarelli (2021). They derive a dynamic, continuous and theory-based measure of the probability of a country to be assigned to different groups. From this analysis they identify three sets of subregions within the Euro area: an extended periphery composed of Finland, Ireland, and Portugal (which is excluded from the analysis); an intermediate group composed of Spain, France, and Italy; and an hard-core group composed of Austria, Belgium, Germany, and Netherlands. Note that Luxembourg is not considered in Campos and Macchiarelli (2021), for this reason it is excluded from the analysis. Another exclusion is that of Portugal, as it is found not to correlate with the other members of the periphery. For simplicity I refer to the first group as "periphery", to the second group as "intermediate", and to the third group as "core".

Following the above clustering, the variables are aggregated through the Principal Component Analysis (PCA). The PCA is an orthogonal linear transformation of the data into a new coordinate system which allows

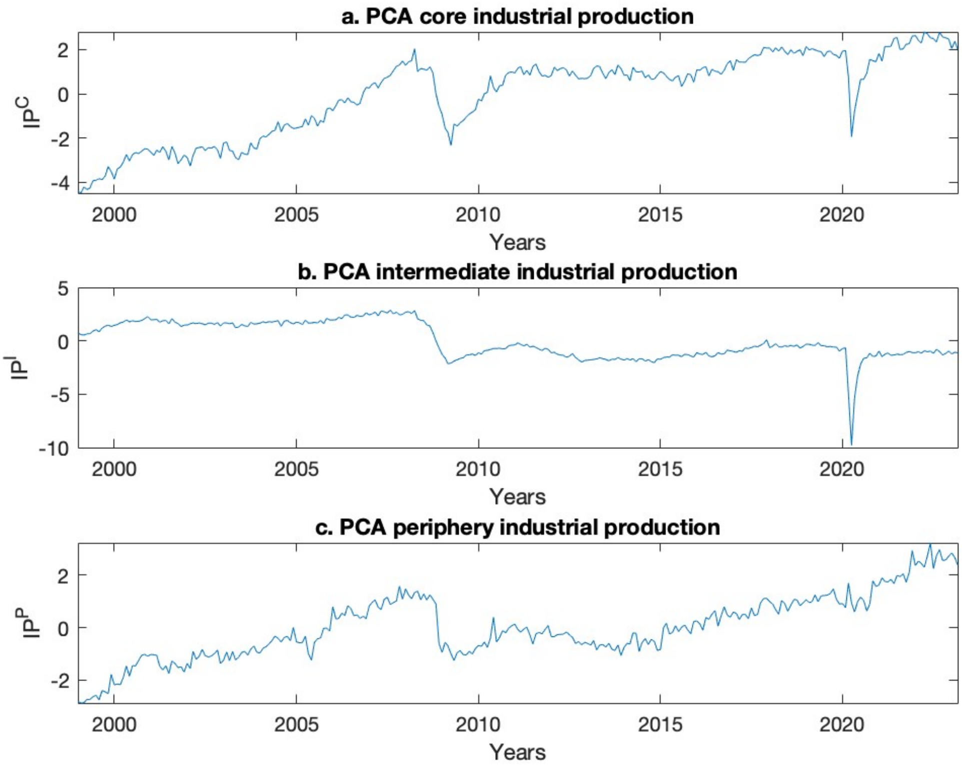

Figure 1.4: plot (from January 1999 to March 2023) of the first component of the PCA of the data on industrial production for the three groups: core in panel a., intermediate in panel b., and periphery in panel c.. Note that this new time series are on a different coordinate system with respect to the original ones, thus the variables are not anymore expressed in the original units.

to condensate a large set of variables into a smaller one that still contains most of the information in the large set. This methodology is used in various fields, including finance, image processing, and genetics. While applying PCA to integrated series might result in some loss of information in the aggregated series, the resulting series closely align with the original ones, as can be seen from the comparison of Figure 1.1, Figure 1.4, Figure 1.2, and Figure 1.5. In Figure 1.4 is presented a plot of the first principal component for industrial production of the three clusters, which represents the aggregated time series for the three clusters of the industrial production variable. Figure 1.5 presents instead the aggregated time series for the unemployment rate variable of the three regions of the Euro area.

Table 1.1: Correlation of national industrial production with the PCA industrial production series.

Country	Core IP	Intermediate IP	Periphery IP
Austria	0.963	-	-
Belgium	0.961	-	-
Germany	0.930	-	-
Netherlands	0.884	-	-
Spain	-	0.973	-
France	-	0.963	-
Italy	-	0.984	-
Finland	-	-	0.865
Ireland	-	-	0.865

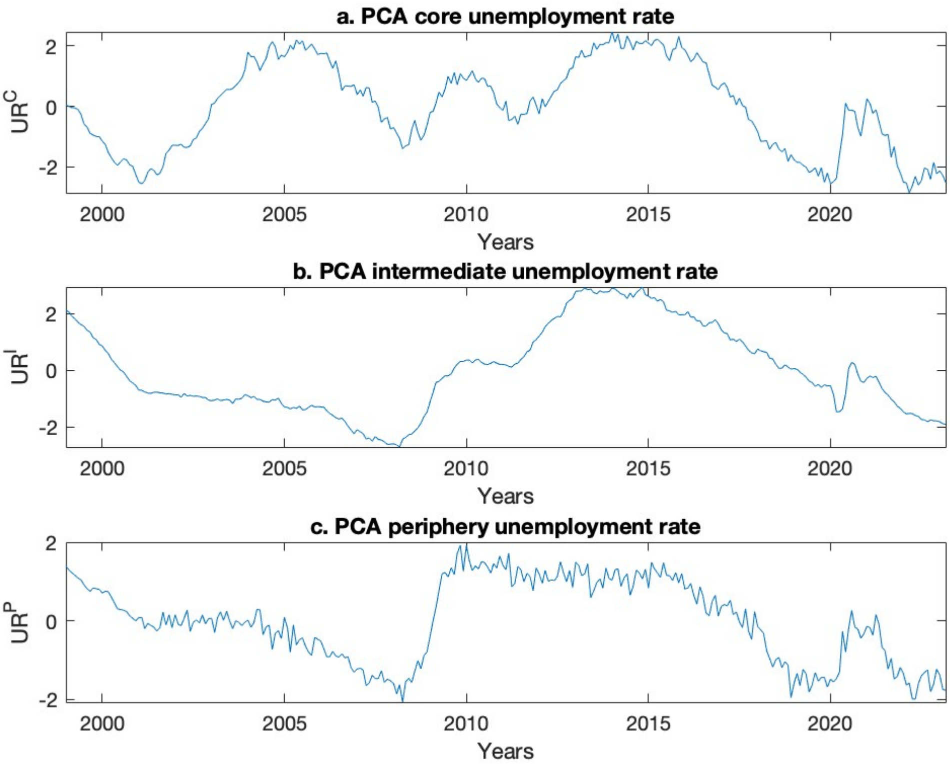

Figure 1.5: plot (from January 1999 to March 2023) of the first component of the PCA of the data on unemployment rate for the three groups: core in panel a., intermediate in panel b., and periphery in panel c.. Note that this new time series are on a different coordinate system with respect to the original ones, thus the variables are not anymore expressed in the original units.

Table 1.2: Correlation of national unemployment rate with the PCA unemployment rate series.

Country	Core UR	Intermediate UR	Periphery UR
Austria	0.677	-	-
Belgium	0.862	-	-
Germany	0.281	-	-
Netherlands	0.919	-	-
Spain	-	0.864	-
France	-	0.854	-
Italy	-	0.926	-
Finland	-	-	0.730
Ireland	-	-	0.730

Table 1.1 and Table 1.2 display the correlation between the initial national time series and the PCA time series of the respective clusters meant to represent them. A high correlation, indicated by values close to 1, indicates that the PCA time series effectively capture the characteristics of the national economies. Table 1.1 and Table 1.2 clearly demonstrate the effectiveness of the clusters from Campos and Macchiarelli (2021), as there is a high correlation between all the economies and the PCA variables. The only exception is the relatively low correlation between the unemployment rate of Germany and the unemployment rate of the core cluster. This suggests that the results derived from the model using PCA variables may not be applicable to the unemployment rate in Germany. Nonetheless, these tables confirm the applicability of the results to all other national economies.

Model

This chapter provides a comprehensive overview of the VECM model used in the analysis. This specific model parameterization is employed to describe the behavior of the variables and capture essential characteristics of a currency union, particularly the enhanced trades and facilitated labor mobility, as emphasized by Mundell (1961), Frankel and Rose (1998), Goodhart (1998), De Grauwe and Mongelli (2005), and the broader OCA theory. Additionally, this chapter proceeds to outline the derivation of the impulse response functions and their corresponding asymptotic distribution.

2.1 Vector Error Correction Model

The model must incorporate specific features of a currency union as well as characteristics pertinent to the analysis. Regarding currency union attributes, a significant aspect underlined by the OCA theory is the presence of multiple economies sharing a common currency and being subject to a single central bank with exclusive authority over monetary policies. Additionally, these economies interact with each other in terms of trades of goods and services and labor mobility, and are all influenced by the same monetary policy stance implemented by the unique central bank (the ECB in the context of the Euro area).

In the context of this analysis, it is relevant for the model to account for long-term relationships among its variables, as suggested by existing literature, including Peersman (2011) and Burriel and Galesi (2018), which identify significant effects persisting for approximately two years following a shock. The VECM parameterization stands out for its ability to handle integrated variables and accommodate the presence of cointegrated variables. This signifies that alterations in the trend of one variable can exert an influence on the trends of other variables, enabling the capture of long-term relationships. As a result, the primary focus is directed towards the original variables' levels rather than their rate of change. Notably, due to the integrated nature of the original variables, the utilization of VAR models is not a feasible approach. VAR models necessitate stationary variables and would entail differencing the time series data. However, adopting such data transformation would disregard the potential for cointegration and result in the loss of critical long-term relationships among the variables. Consequently, VAR models are typically reserved for the examination of short-term relationships among variables.

The VECM offers a convenient way to parameterize all of these aforementioned features of interest. Specifically, it enables the handling of cointegrated variables as well as the enhanced interactions among member states in terms of production and labor mobility, and the presence of a unique central bank affecting all member states through its monetary policy actions. The VECM model addresses long-term relations by considering the possibility of multiple equilibria among the variables. Any deviations from these equilibria are introduced exogenously into the model, a characteristic denoted by the "error correction" term in its name. This feature allows for the representation of long-term relationships among the variables.

The VECM($p-1$) model has the following structure

$$\Delta y_t = \alpha\beta' y_{t-1} + \sum_{i=1}^{p-1} \Gamma_i \Delta y_{t-i} + u_t \ , \tag{2.1}$$

where

$$y_t = \begin{bmatrix} IP_t^C & IP_t^I & IP_t^P & UR_t^C & UR_t^I & UR_t^P & MP_t \end{bmatrix}' .$$

y_t is a $K \times 1$ vector, with K being the number of variables ($K = 7$ in the context of this model), IP_t^C and UR_t^C represent industrial production and unemployment rate for the core economies, IP_t^I and UR_t^I being those of the intermediate economies, IP_t^P and UR_t^P those of the periphery, and MP_t the monetary policy variable. α and β are $K \times r$ matrices of rank r, where r is the number of common trends in the variables (which also represent the number of equilibrium). I derive the optimal value of r by implementing the Johansen cointegration test. The Γ_i are the $K \times K$ autoregressive coefficients, and p comes from the VAR(p) model corresponding to the VECM($p-1$). The optimal lag order p is chosen starting from a VAR model, and selecting the value for p minimizing the Akaike information criteria (or alternatively the Bayesian information criteria). u_t is the $K \times 1$ white noise vector, which is normally distributed with mean zero and variance Σ_u. β is referred to as the matrix of cointegration, and is responsible to make the variable y_{t-1} stationary and, even more importantly, is responsible of capturing deviations from equilibrium of the variables in y_{t-1}. α represents the coefficient with which the variables respond to deviations from equilibrium.

To highlight the specific features of the parameterization in (2.1), suppose to have two variables only

$$y_t = \begin{bmatrix} y_{1,t} & y_{2,t} \end{bmatrix}' .$$

Suppose also that there is only one equilibrium condition, which is satisfied for $y_{1,t} = \beta_1 y_{2,t}$ (note that in this specific case $r = 1$, i.e. there is only one equilibrium condition or the cointegration rank is one), then a VECM(1) model can be explicitly expressed as follows

$$\Delta y_{1,t} = \alpha_1(y_{1,t-1} - \beta_1 y_{2,t-1}) + \gamma_{11,1}\Delta y_{1,t-1} + \gamma_{12,1}\Delta y_{2,t-1} + u_{1,t}$$

$$\Delta y_{2,t} = \alpha_2(y_{1,t-1} - \beta_1 y_{2,t-1}) + \gamma_{21,1}\Delta y_{1,t-1} + \gamma_{22,1}\Delta y_{2,t-1} + u_{2,t} \ .$$

The important feature to be noted is the role of the $K \times 1$ β_1 vector, where $r = 1$ is number of common trends and, as said, plays an important double role in this parameterization. Firstly, it transforms the y_{t-1} variable into a stationary term. Secondly, it captures the deviation from the equilibrium, where this deviation is given by the $(y_{1,t-1} - \beta_1 y_{2,t-1})$ term. Finally from the $\alpha_1(y_{1,t-1} - \beta_1 y_{2,t-1})$ (or analogously from the $\alpha_2(y_{1,t-1} - \beta_1 y_{2,t-1})$) term, the role of the α_i's coefficients becomes clear, they capture the impact that a deviation from equilibrium has on the variables of the model.

The fact that this parameterization allows to consider various equilibrium (the number of equilibrium is given by the rank r of the α and β matrices) enables the model to capture the new Keynesian idea of existence of an equilibrium for unemployment, without having to limit the number of equilibrium conditions to this variable only. The only requirement for the model to be correctly specified is $r < K$, thus there have to be fewer equilibrium than there are variables. The speed at which the variables adjust is left to be determined by the data[1].

2.2 Impulse Response Function

The ultimate goal of this analysis is to estimate the influence of an unforeseen monetary policy event on the previously described model. The concept here is to examine how the endogenous variables within the system evolve over time when one variable experiences a shock at a specific point in time, denoted as $t = 0$. This study focuses on understanding the dynamics of these variables in response to such an innovation, which is defined as derivation of the impulse response functions (IRF).

In a stable VAR model, the impulse response functions are derived from the Moving Average (MA) representation of the process. Due to the stability of the process, these impulse responses tend to approach zero over time. In other words, the impact of an innovation is typically short-lived, capturing primarily the short-term relationships among the variables when the innovation occurs within the system. However, I aim to avoid confining the analysis solely to these short-term relationships. As previously mentioned, long-term relationships can be of significant importance. This consideration underscores one of the key rationales behind opting for a VECM parameterization.

In a VECM model, some of the variables are integrated and may even be cointegrated. Consequently, the VECM process is non-stationary, and the impulse responses that will be derived may not necessarily diminish over time. In other words, these impulse responses may not necessarily converge towards zero over time, as would be the case in a stationary process. It's important to note that this implies that the analysis enables an exploration of whether the model eventually reaches a new equilibrium over time. Moreover, it provides insights into the levels of the variables in this new equilibrium compared to the state prior to the occurrence of the innovation.

An unstable integrated or cointegrated process does not posses a valid MA representation. However, the impulse responses of the VECM can be derived in a similar way to what is done for the stable VAR processes.

[1] One of the key distinctions between the perspectives of new classical economics and new Keynesian economics lies in their views on the speed of variable adjustment following a shock. The new classical school contends that variables adjust instantaneously, whereas the new Keynesians maintain that prices are sticky, leading to a gradual adjustment of variables over a certain period following the innovation.

The VECM($p-1$) model can be represented through a VAR(p) parameterization. Consider (2.1), reported below for convenience

$$\Delta y_t = \alpha\beta' y_{t-1} + \textstyle\sum_{i=1}^{p-1} \Gamma_i \Delta y_{t-i} + u_t \ ,$$

by noting that $\Delta y_t = y_t - y_{t-1}$, the above can be rewritten as

$$y_t - y_{t-1} = \alpha\beta' y_{t-1} + \Gamma_1(y_{t-1} - y_{t-2}) + \cdots + \Gamma_{p-1}(y_{t-p+1} - y_{t-p}) + u_t \ .$$

By grouping the common variables y_{t-i}, for i = 1,2,...,p

$$y_t = (\alpha\beta' + I_K + \Gamma_1)y_{t-1} + (\Gamma_2 - \Gamma_1)y_{t-2} + \cdots + (\Gamma_{p-1} - \Gamma_{p-2})y_{t-p+1} - \Gamma_{p-1}y_{t-p} + u_t \ ,$$

I can express the VECM(p-1) as a VAR(p)

$$y_t = \sum_{i=1}^{p} A_i y_{t-1} + u_t \ , \tag{2.2}$$

where the A_i are

$$A_1 = \alpha\beta' + I_K + \Gamma_1 \ ,$$

$$A_i = \Gamma_i - \Gamma_{i-1}, \quad \text{for } i = 2,...,p \text{ - } 1 \ ,$$

$$A_p = -\Gamma_{p-1} \ ,$$

and I_K is the $K \times K$ identity matrix.

Given the VAR(p) formulation in (2.2), the impulse response function is derived as follows. I rewrite the VAR(p) into a VAR(1) and obtain

$$Y_t = \mathbf{A} Y_{t-1} + U_t \ , \tag{2.3}$$

where:

$$Y_t = \begin{bmatrix} y_t \\ \vdots \\ y_{t-p+1} \end{bmatrix},$$

is a $Kp \times 1$ vector of endogenous variables;

$$\boldsymbol{A} = \begin{bmatrix} A_1 & A_2 & \dots & A_{p-1} & A_p \\ I_K & 0 & \dots & 0 & 0 \\ 0 & I_K & & 0 & 0 \\ \vdots & & \ddots & \vdots & \vdots \\ 0 & 0 & \dots & I_K & 0 \end{bmatrix},$$

is a $Kp \times Kp$ matrix of coefficients; and

$$U_t = \begin{bmatrix} u_t \\ 0 \\ \vdots \\ 0 \end{bmatrix},$$

is a $Kp \times 1$ vector of residuals, where u_t is white noise with Σ_u variance ($u_t \sim N(0, \Sigma_u)$). The $K \times K$ MA coefficient matrices are derived as follows

$$\Phi_0 = I_K \ ,$$

$$\Phi_i = J\boldsymbol{A}^i J', \quad \text{for } i = 1,2,\dots \ ,$$

where $J = \begin{bmatrix} I_K & 0 & \dots & 0 \end{bmatrix}$ is a $K \times Kp$ matrix. In a stable VAR model, these matrices allow to rewrite the process in its MA representation

$$y_t = \sum_{i=0}^{\infty} \Phi_i u_{t-i} \ . \tag{2.4}$$

The Φ_is in (2.4) are $K \times K$ matrices that represent the impact of an error term u_{t-i}, occurring i periods in the past, on the variables in y_t at period t.

It's essential to emphasize that the Φ_i matrices in (2.4) are not orthogonal, implying potential correlations among error terms across different variables. Consequently, when employing these Φ_i coefficients for impulse response analysis, a shock to one variable at $t = 0$ can have a contemporaneous effect on other variables. Given that the ECB independently formulates its monetary policies, and considering the stickiness in the response of national economies to these shocks, as demonstrated by Golosov and Lucas Jr. (2007), it becomes reasonable to consider employing orthogonal MA coefficients. These orthogonal coefficients ensure that the shock to one variable in the model at $t = 0$ influences the other variables in the system with some delay, starting no earlier than $t = 1$.

To achieve this, I utilize the Choleski decomposition of Σ_u, expressed as $\Sigma_u = PP'$, where P is a lower triangular matrix with dimensions $K \times K$. I then use this matrix P to construct the orthogonal impulse responses (Θ_i). The choice of a lower triangular P matrix aligns with the placement of the monetary policy variable at the bottom of the y_t vector of system variables. Under these specifications, the derivation of the orthogonal MA coefficients in (2.6) ensures that the innovation in monetary policy at $t = 0$ impacts the other variables in the system with some delay, commencing at least at $t = 1$. To obtain these $K \times K$ orthogonal coefficients, I perform matrix multiplication of the right-hand side of (2.4) with P and the inverse of P (i.e., P^{-1})[2]

$$y_t = \sum_{i=0}^{\infty} \Phi_i P P^{-1} u_{t-i} \ .$$

I define $\Theta_i = \Phi_i P$, where Θ_i is the orthogonal MA matrix of coefficients, and $w_t = P^{-1} u_{t-i}$. Since $u_t \sim N(0, \Sigma_u)$, and $\Sigma_u = PP'$, then $P^{-1} u_{t-i} = w_t \sim N(0, I_K)$[3]. Note from this last derivation that the error terms in w_t are now uncorrelated. Thus shocking one variable at $t = 0$ does not affect the other variables before $t = 1$. This reformulation allows to write the original process of y_t through an orthogonal MA representation

$$y_t = \sum_{i=0}^{\infty} \Theta_i w_{t-i} \ . \tag{2.5}$$

Thus, the orthogonal MA coefficients adopted to derive the impulse response function are

$$\Theta_i = \Phi_i P, \quad \text{for } i = 0,1,2,\ldots \ , \tag{2.6}$$

which can be computed also in the case of a non-stationary VECM model, although it does not posses any proper MA representation of its process. The jk-th element of Θ_i is assumed to represent the impact on variable j of a standard deviation innovation in variable k, which has occurred i periods in the past. Thus, the impulse response function of a shock to monetary policy (variable seven in the system) on variable j for $j = 1,2,\ldots,K$ is obtained by collecting the $j7$-th element of Θ_i, for $i = 0,1,2,\ldots$, into a single vector. More in general, the impulse response function of variable j to a one standard deviation innovation in variable k, namely IRF_{jk}, is computed as

$$IRF_{jk} = \begin{bmatrix} \Theta_{0,jk} & \Theta_{1,jk} & \Theta_{2,jk} & \ldots & \Theta_{n,jk} \end{bmatrix} \ . \tag{2.7}$$

2.3 Asymptotic Distribution of Impulse Response Function

The confidence interval for the impulse response function is build adopting its asymptotic distribution. The distribution is derived as in "New Introduction to Multiple Time Series Analysis" by Lutkepohl.

Consider the following notation:

$$\boldsymbol{\alpha} = vec(A_1, \ldots, A_p) \ ,$$

where $\boldsymbol{\alpha}$ is a $K^2 p \times 1$ vector, and vec is an operator that transforms a matrix into a column vector by vertically

[2] Note that $PP^{-1} = I_K$, and any $K \times K$ matrix A multiplying I_K gives back the matrix A, $AI_K = A$.

[3] Note that $P^{-1} u_{t-i} = P^{-1} N(0, \Sigma_u) = N(0, P^{-1} \Sigma_u (P^{-1})')$. Since $P^{-1} \Sigma_u (P^{-1})' = P^{-1} PP' (P^{-1})' = I_K$ given that $\Sigma_u = PP'$, then $w_t \sim N(0, I_K)$.

stacking the columns of the matrix;

$$\boldsymbol{\sigma} = vech(\Sigma_u) \ ,$$

where $\boldsymbol{\sigma}$ is a $(1/2)K(K+1) \times 1$ vector, and *vech* is an operator that takes only the elements on and below the main diagonal of a matrix, and vertically stacks them into a column vector.

Then, suppose:

$$\sqrt{T} \begin{bmatrix} \hat{\boldsymbol{\alpha}} - \boldsymbol{\alpha} \\ \hat{\boldsymbol{\sigma}} - \boldsymbol{\sigma} \end{bmatrix} \xrightarrow{d} N(\, 0, \begin{bmatrix} \Sigma_{\hat{\boldsymbol{\alpha}}} & 0 \\ 0 & \Sigma_{\hat{\boldsymbol{\sigma}}} \end{bmatrix}) \ , \tag{2.8}$$

where $\hat{\boldsymbol{\alpha}}$ represents the vectorized maximum likelihood estimates of the $A_1, ..., A_p$ autoregressive matrices, $\hat{\boldsymbol{\sigma}} = vech(\hat{\Sigma}_u)$ where $\hat{\Sigma}_u = (\frac{1}{T-p-Kp})(\hat{u}\hat{u}') = (\frac{1}{T-p-Kp})(Y - \hat{\boldsymbol{A}}X)(Y - \hat{\boldsymbol{A}}X)'$, and

$$\hat{\Sigma}_{\hat{\boldsymbol{\alpha}}} = (XX')^{-1} \otimes ((Y - \hat{\boldsymbol{A}}X)(Y - \hat{\boldsymbol{A}}X)') \ , \tag{2.9}$$

with $\otimes$ being the Kronecker product, and

$$Y = \begin{bmatrix} y_1 & \dots & y_T \end{bmatrix} \ ,$$

$$Y_{t-1} = \begin{bmatrix} y_{t-1} \\ \vdots \\ y_{t-p} \end{bmatrix} \ ,$$

$$X = \begin{bmatrix} Y_0 & \dots & Y_{t-1} \end{bmatrix} \ .$$

Y is the $K \times T$ matrix of endogenous variables, Y_{t-1} is a $K^2p \times 1$ vector, and X is the $K^2p \times T$ matrix of exogenous variables, with T being the sample size; and

$$\hat{\Sigma}_{\hat{\boldsymbol{\sigma}}} = 2D_K^+(\hat{\Sigma}_u \otimes \hat{\Sigma}_u)D_K^+, \tag{2.10}$$

with D_K^+ being the Moore Penrose generalized inverse of the duplication matrix D_K.

Then the asymptotic distribution of Θ_i is

$$\sqrt{T}\ vec\ (\hat{\Theta}_i - \Theta_i) \xrightarrow{d} N(\, 0,\ C_i \Sigma_{\hat{\alpha}} C_i' + \bar{C}_i \Sigma_{\hat{\sigma}} \bar{C}_i'\ ), \quad \text{for } i = 0,1,2,... \ , \tag{2.11}$$

with

$$C_0 = 0 \ ,$$

$$C_i = (P' \otimes I_K)G_i, \quad \text{for } i = 1,2,... \ ,$$

$G_i = \sum_{m=0}^{i-1} J(\boldsymbol{A}')^{i-1-m} \otimes \Phi_m, \quad \text{for } i = 0,1,2,...$,

$\bar{C}_i = (I_K \otimes \Phi_i)H, \quad \text{for } i = 0,1,2,...$,

$H = L_K'(L_K(I(K^2) + K_{KK})(P \otimes I_K)L_K')^{-1}$.

L_K is the $K^2 \times (1/2)K(K+1)$ elimination matrix, for which $vech(A) = L_K vec(A)$, with A being $K \times K$; and K_{KK} is the $K \times K$ commutation matrix such that $vec(A) = K_{KK} vec(A')$.

Finally the confidence interval of Θ_i at the α% significance level is computed as

$$CI_{\alpha,\hat{\Theta}_i} = vec(\hat{\Theta}_i) \pm z_{\alpha/2}((T)^{-1}\sqrt{diag(Var(\Theta_i))}) \ , \tag{2.12}$$

where $Var(\Theta_i)$ is the variance in (2.11); the *diag* operator takes the diagonal elements of the matrix and stacks them into a column vector; and $z_{\alpha/2}$ is the inverse of the standard normal cumulative distribution function (cdf), evaluated at the probability $(1 - \alpha/2)$.

2.4 Maximum Likelihood Estimators of a VECM

Consider rewriting the VECM in (2.1) for $t = 1,2,...,T$ in matrix notation

$$\Delta Y = \Pi Y_{-1} + \Gamma \Delta X + U. \tag{2.13}$$

where

$\Pi = \alpha\beta'$,

$\Delta Y = \begin{bmatrix} \Delta y_1 & \dots & \Delta y_T \end{bmatrix}$,

$Y_{-1} = \begin{bmatrix} y_0 & \dots & y_{T-1} \end{bmatrix}$,

$\Gamma = \begin{bmatrix} \Gamma_1 & \dots & \Gamma_{p-1} \end{bmatrix}$,

$\Delta X = \begin{bmatrix} \Delta X_0 & \dots & \Delta X_{T-1} \end{bmatrix}$, with

$\Delta X_{t-1} = \begin{bmatrix} \Delta y_{t-1} \\ \vdots \\ \Delta y_{t-p+1} \end{bmatrix}$, and

$U = \begin{bmatrix} u_1 & \dots & u_T \end{bmatrix}$.

By assuming that the y_t process is Gaussian (or normally distributed), with $u_t \sim N(0, \Sigma_u)$, then the VECM in (2.1) can be estimated by Maximum Likelihood (ML) by taking also the rank restriction for $\Pi = \alpha\beta'$ into account (Johansen (1998) and Johansen (1995)). The log-likelihood function is

$$\ln l = -\frac{1}{2} tr[(\Delta Y - \alpha\beta' Y_{-1} - \Gamma\Delta X)' \Sigma_u^{-1} (\Delta Y - \alpha\beta' Y_{-1} - \Gamma\Delta X)]. \tag{2.14}$$

Defining the following elements

$M = I_T - \Delta X'(\Delta X \Delta X')^{-1}\Delta X$,

$R_0 = \Delta Y M$,

$R_1 = Y_{-1} M$,

$S_{ij} = \frac{1}{T} R_i R_j'$, for i, j = 0,1,

$\lambda_1 \geq \cdots \geq \lambda_K$ are the eigenvalues of

$S_{11}^{-1/2} S_{10} S_{00}^{-1} S_{01} S_{11}^{-1/2}$, and

$\nu_1, \ldots, \nu_K$ are the corresponding orthonormal eigenvectors,

the log-likelihood function in (2.14) is maximized for the following ML parameters

$\beta = \tilde{\beta} = \begin{bmatrix} \nu_1 & \ldots & \nu_K \end{bmatrix}' S_{11}^{-1/2}$,

$\alpha = \tilde{\alpha} = \Delta Y M Y_{-1}' \tilde{\beta}' (\tilde{\beta}' Y_{-1} M Y_{-1}' \tilde{\beta})^{-1} = S_{01} \tilde{\beta} (\tilde{\beta}' S_{11} \tilde{\beta})^{-1}$,

$\Gamma = \tilde{\Gamma} = (\Delta Y - \tilde{\alpha}\tilde{\beta}' Y_{-1}) \Delta X' (\Delta X \Delta X')^{-1}$,

$\Sigma_u = \tilde{\Sigma}_u = \frac{1}{T} (\Delta Y - \tilde{\alpha}\tilde{\beta}' Y_{-1} - \tilde{\Gamma}\Delta X)(\Delta Y - \tilde{\alpha}\tilde{\beta}' Y_{-1} - \tilde{\Gamma}\Delta X)'$.

Results

This chapter presents the results obtained from the analysis, encompassing three distinct estimations that utilize the three monetary policy instruments: the policy rate or interest rate, the monetary aggregate M2, and the monetary aggregate M3. The overarching objective of this analysis is to ascertain whether there exists heterogeneity across the Euro area concerning responses to unforeseen monetary policy actions by the ECB. Consequently, the ensuing results depict the impulse response functions of the system's variables in response to a shock in the monetary policy instrument.

3.1 Policy Rate Impulse Responses

In this analysis the variable used as indicator of the ECB's monetary policy is the interest rate (ir), proxied by the minimum bid rate on the ECB's main refinancing operations, as explained in Chapter 1. Thus the variables of the model are

$$y_t = \begin{bmatrix} IP_t^C & IP_t^I & IP_t^P & UR_t^C & UR_t^I & UR_t^P & ir_t \end{bmatrix}'.$$

By utilizing these variables, the Johansen cointegration test suggests a cointegration rank of five ($r = 2$), indicating the presence of two distinct equilibrium conditions. Given that r is both greater than 0 and less than K, it is feasible to estimate the VECM model. To determine the optimal lag, the Akaike information criteria is employed, suggesting a lag of $p = 3$. Consequently, with the specified variables, a VECM(2) model with a cointegration rank of $r = 2$ is estimated by maximum likelihood. The details on the estimated parameters of the model, as well as a check on the assumption of Gaussianity and independence of the residuals can be found in Appendix I. From this estimated model, impulse response functions for the seven variables in response to a one standard deviation innovation in the interest rate are retrieved, as outlined in Section 2.2. The asymptotic confidence intervals, constructed as described in Section 2.3, offer confidence bands that are somewhat less precise (though still in line) compared to the bootstrap confidence intervals. This is due to the limited sample size available. Therefore, a 68% bootstrap confidence interval is adopted for the analysis. The estimated impulse response functions are presented in Figure 3.1 for the three industrial production variables and in Figure 3.2 for the three unemployment rate variables along with the interest rate.

Figure 3.1 illustrates how industrial production initially decreases for all three macro-economies in the months following an expansionary monetary policy. Industrial production in both the core (Austria, Belgium, Germany, and the Netherlands) and intermediate (Spain, France, and Italy) economies, as depicted in Figure 3.1a and Figure 3.1b respectively, shows a rebound to zero levels after approximately one year. This indicates that the ECB's expansionary monetary policy, enacted through the lowering of the interest rate, fails to stimulates these economies.

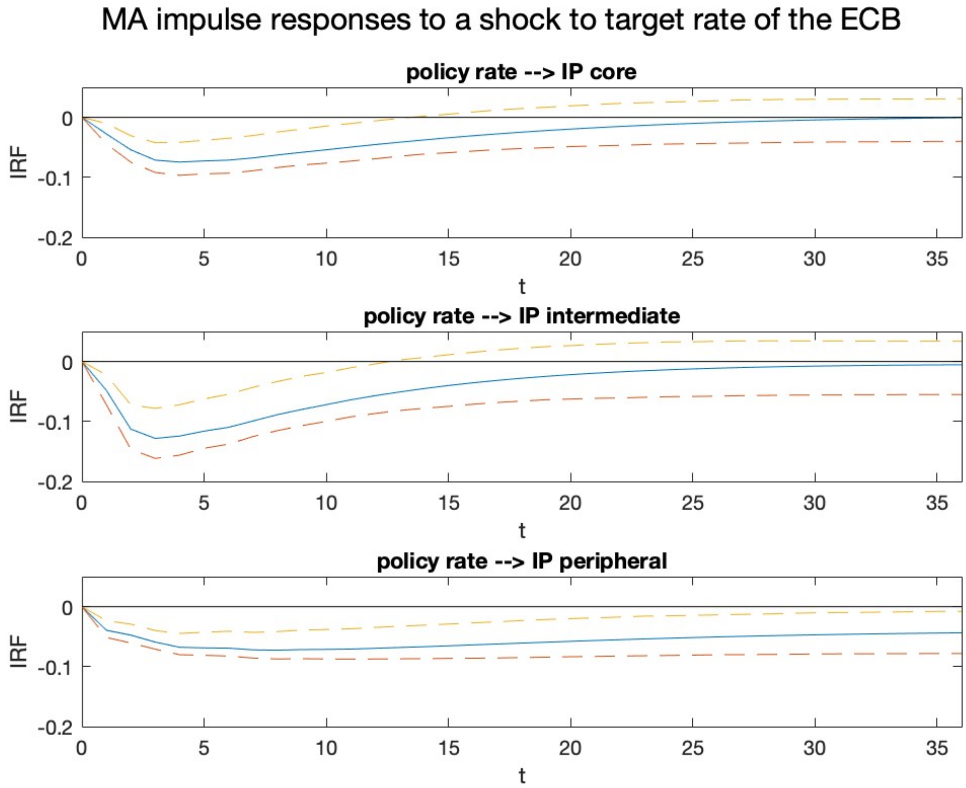

Figure 3.1: impulse response functions of industrial production for the core (panel a.), intermediate (panel b.), and peripheral (panel c.) aggregated economies, to a one standard deviation innovation in the interest rate of the ECB. The "t" variable represents the months, and the zero indicates the moment in which the ECB adopts its unexpected expansionary monetary policy, i.e. the moment in which it unexpectedly decides to lower the interest rates. The dotted lines represent the upper and lower bounds of the 68% bootstrap confidence intervals for the estimated impulse responses. The blue solid lines represent the expected value of the impulse response functions. The impulse response functions for the remaining variables are reported in Figure 3.2.

Turning to the peripheral economies (Finland and Ireland), as depicted in Figure 3.1c, the expansionary monetary policy appears to lack the desired effect, with industrial production ultimately stabilizing statistically below zero in the long-run. Moreover, intermediate economies exhibit a larger short-term decline in industrial production compared to both core and peripheral economies. The initial indication of heterogeneity in responses suggests that certain economies do not exhibit the same reactions as the other Euro area economies analyzed in this study. Specifically, Finland and Ireland experience their industrial production stabilizing below its initial level, while France, Spain and Italy face a more significant short-term drop in industrial production compared to the other two macro areas. Furthermore, it's noteworthy that these economies do not respond to interest rate stimuli through the unemployment rate channel. Peersman (2011) conducted an analysis of the impact of changes in the interest rate on Euro area economies, specifically through the credit channel. His findings indicate a similar short-term drop but a statistically significant rebound in the longer run. Therefore, it appears that the stimulus primarily arises from the credit channel rather than the employment channel, which contrasts with the new Keynesian view. This phenomenon of heterogeneity, in line with the OCA theory, hinders the progress of the Euro area toward becoming an optimal currency area and exacerbates the economic challenges faced by the participating economies within the currency union.

Regarding the remaining variables within the model, Figure 3.2 illustrates their impulse responses. In this case as well, there is evidence of heterogeneity across the three macro-areas. The intermediate and peripheral economies, as shown in Figure 3.2b and Figure 3.2c, exhibit no significant effects on their unemployment rates, except for a minor short-term increase, corresponding to the reduced production. They tend to readjust and stabilize at their pre-shock levels. In contrast, the impact of monetary policy on the core economies' unemployment rate, as observed in Figure 3.2a, is statistically significant and stabilizes above zero.

Of particular interest is the behavior of the monetary policy variable following its unexpected change, as illustrated in Figure 3.2d. Initially, it continues to decrease for the first seven to ten months following the expansionary policy action, before stabilizing at a new level slightly below zero, close to the post-shock level. This pattern aligns with the understanding that central banks must take into account agents' expectations when implementing their policies. Frequent shifts in monetary policy stance (from expansionary to contractionary, or vice versa) could erode trust in their policies, potentially undermining their control over the economy. This aspect gained prominence during the European sovereign debt crisis, prompting the ECB to introduce "forward guidance," in which they provide information about their anticipated future policy actions and their macroeconomic assessments. It's also relevant to highlight that the interest rate remaining permanently at the new lower level may pose challenges when the policy rate is close to zero, as it limits the margin the ECB has for further rate reductions in the longer run. Based on this result, when policy rates approach zero, it becomes necessary for the ECB to implement unconventional policies in order to have an impact on the economy.

Given that the ECB avoids unexpected monetary policy actions, one could argue that the whole analysis of these unexpected policies is irrelevant. While the significance of forward guidance and the central banks' reluctance to implement unexpected monetary policy actions are evident under normal conditions, it is important to note that they may no longer hold true in the face of unforeseen major events, such as the COVID-19 pandemic or the conflict between Ukraine and Russia. During such crises, expectations are less likely to be met due to heightened environmental uncertainty. It is in these situations that this analysis becomes particularly relevant.

Analyzing Figures 3.1 and 3.2, it becomes evident that there is an initial reaction (which can be considered a short-term response) to the monetary policy within the first ten months following the innovation. Only in the longer run do the variables adjust to similar equilibrium in which: industrial production stabilizes at pre-innovation level for the core and intermediate economies but below for the peripheral economies; the unemployment rate for the intermediate and peripheral economies equilibrium at the same level prior the innovation but remains above it for the core economies; the interest rate permanently remains below its pre-shock level, and close to the level set by the shock.

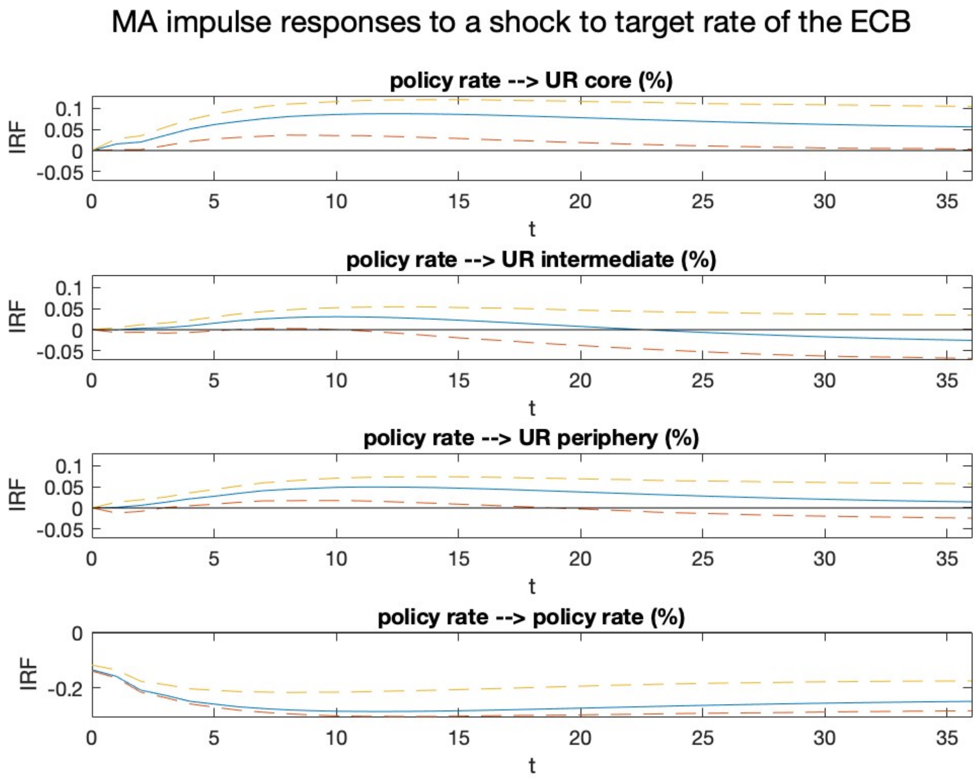

Figure 3.2: impulse response functions of unemployment rate for the core (panel a.), intermediate (panel b.), and peripheral (panel c.) aggregated economies, and of the interest rate (panel d.), to a one standard deviation innovation in the interest rate of the ECB. The "t" variable represents the months, and the zero indicates the moment in which the ECB adopts its unexpected expansionary monetary policy, i.e. the moment in which it unexpectedly decides to lower the interest rates. The dotted lines represent the upper and lower bounds of the 68% bootstrap confidence intervals for the estimated impulse responses. The blue solid lines represent the expected value of the impulse response functions.

A noteworthy observation requires further investigation, although it lies beyond the scope of this study. The unemployment rate channel, considered a key aspect of an optimal currency area as emphasized by Mundell (1961) and others, does not exhibit a response to interest rate actions implemented by the ECB. Consequently, industrial production is not stimulated through this channel. Peersman (2011) analyzes the impulse responses for the Euro area to an innovation in interest rates, examining the effects through the credit channel and finding positive long-term impacts on the Euro area's economic performance. Hence, exploring the heterogeneity of Euro area economies' responses to interest rate innovations while considering the credit channel may warrant further investigation.

3.2 M2 Impulse Responses

In this analysis, the variable chosen to represent the ECB's monetary policy is the monetary aggregate M2[1]. This variable serves as an indicator of unconventional monetary policy measures. Burriel and Galesi (2018) conducted an analysis of the impact of unconventional monetary policies within the Euro area context, revealing that such policies have a stabilizing effect on economies and lead to improved economic performance. It is essential to

[1] M2, as defined by the ECB, encompasses the sum of currency in circulation, overnight deposits, deposits with an agreed maturity of up to two years, and deposits redeemable at notice of up to three months.

note that, in accordance with the new Keynesian theory, alterations in the money supply exert an indirect influence on the real economy, with this effect transmitted through interest rates. Viewing the interest rate as the cost of money, it becomes apparent that an increase in the money supply (while maintaining demand constant) results in a lower interest rate. This explanation aligns with the Keynesian perspective of the causal chain. Consequently, an expansionary monetary policy, aimed at stimulating the economy, can be implemented by increasing the money supply, as represented by the monetary aggregate M2 in this context.

The variables of this model are thus the following

$$y_t = \begin{bmatrix} IP_t^C & IP_t^I & IP_t^P & UR_t^C & UR_t^I & UR_t^P & M2_t \end{bmatrix}'.$$

The Johansen cointegration test conducted on these variables suggests an optimal cointegration rank of four ($r = 4$). The Akaike information criteria suggest an optimal lag of three ($p = 3$), while the Bayesian information criteria recommend an optimal lag of one ($p = 1$). Given that a VECM(0) would solely incorporate the error correction term $\alpha\beta' y_{t-1}$ from the right-hand side of (2.1), representing the deviation from equilibrium, and no lagged terms, I opt to follow the suggestion of the Akaike information criteria. Thus, for this specific analysis, I estimate a VECM(2) with a cointegration rank of four by maximum likelihood. The details on the estimated parameters of the model, as well as a check on the assumption of Gaussianity and independence of the residuals can be found in Appendix II.

Figure 1.3b illustrates that there is relatively little variance in the stock of M2. Consequently, the asymptotic variance of the orthogonal MA coefficients (the Θ_i's) for monetary policy becomes very small. Using the asymptotic distribution to estimate confidence intervals would yield bounds that are flattened on the expected value. To address this issue, I adopt bootstrap (68%) confidence intervals for this analysis, providing more realistic results[2].

Similar to the previous section, Figure 3.3 displays the estimated impulse response functions for industrial production in the three macro-areas in response to an unexpected one standard deviation innovation in the stock of M2, signifying an expansionary monetary policy action by the ECB. Additionally, Figure 3.4 presents the impulse response functions for the remaining variables in the model, including the unemployment rate of the three economies and the stock of M2.

Industrial production variables adjust to new higher levels for the core (Austria, Belgium, Germany, and Netherlands) and peripheral (Finland and Ireland) economies, as can be seen from Figure 3.3a and Figure 3.3c. There are differences between the responses of these two macro-areas, with respect to what has been found in Section 3.1. The peripheral economies' industrial production respond to the expansionary innovation without falling statistically below zero in the short-run, and stabilizing statistically above zero. Furthermore, the core economies are stimulated in production by this policy. The response of the intermediate economies

[2] The bootstrapped 68% confidence intervals estimated for the model in Section 3.1 are similar to the asymptotic ones, even if more precise. The main difference is that in that case, the bounds estimated by adopting the asymptotic distribution of the Θ_i's are symmetric with respect to the expected value. Achieving similar results in the confidence intervals for that analysis was possible due to the larger variance in interest rates compared to the stock of M2, leading to larger variances for the impulse response functions. One potential reason for the stock of M2 exhibiting such little variance may be the ECB's limited changes to M2 and the fact that the variable's variance is further reduced by taking its natural logarithm.

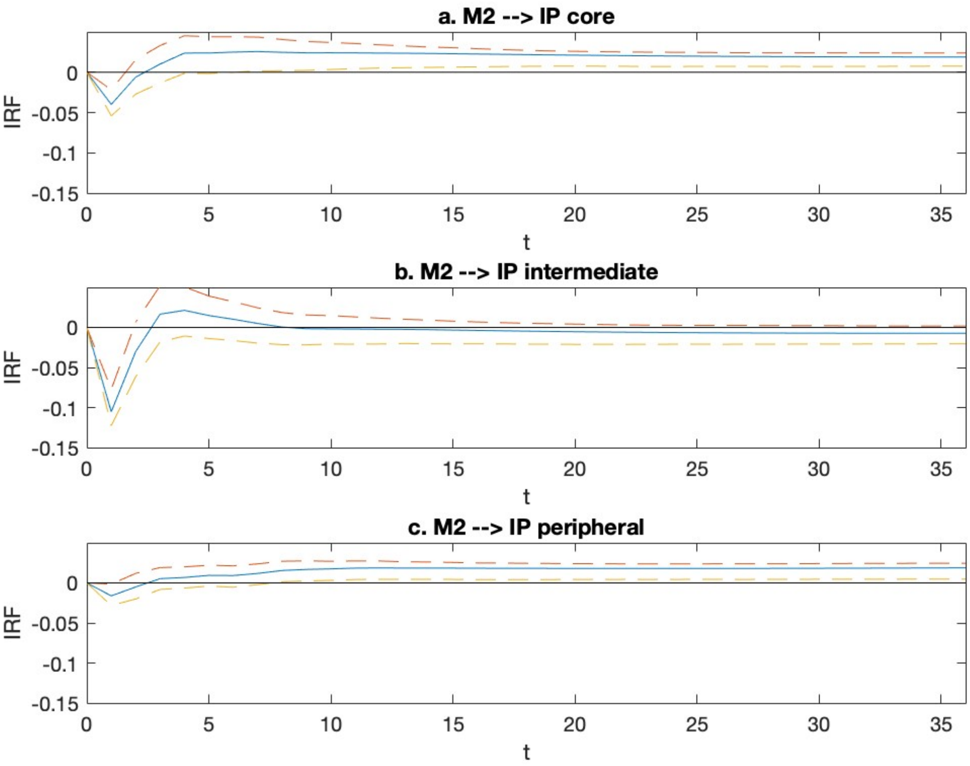

Figure 3.3: impulse response functions of industrial production for the core (panel a.), intermediate (panel b.), and peripheral (panel c.) aggregated economies, to a one standard deviation innovation in the stock of M2 of the ECB. The "t" variable represents the months, and the zero indicates the moment in which the ECB adopts its unexpected expansionary monetary policy, i.e. the moment in which it unexpectedly decides to increase the stock of M2. The dotted lines represent the upper and lower bounds of the 68% bootstrap confidence intervals for the estimated impulse responses. The blue solid lines represent the expected value of the impulse response functions. The impulse responses for the remaining variables of the system can be found in Figure 3.4.

instead is in line with the previous analysis: industrial production drops in the first few months following the innovation, before re-bouncing and stabilizing at statistically zero values, thus levels equal to those prior the innovation(Figure 3.3b). The expansionary monetary policy action adopted by the ECB has no expansionary effect on output for these economies. The risk of undertaking an expansionary monetary policy through the increase in money supply by increasing the stock of M2, is that the intermediate economies are left back in terms of production with respect to the rest of the Euro area economies considered. This results in higher "costs" of belonging to a monetary union for the member states.

The response of unemployment rate for the three macro-areas is similar with respect to what has been found in section 3.1. Unemployment rate for the core, intermediate and peripheral macro-areas (Figure 3.4a, Figure 3.4b, and Figure 3.4c) remain statistically at zero (except for the core economies that see their unemployment rate slightly raise for the first few months following the innovation). This result provides evidence of the fact that also monetary policies acting through the monetary aggregate M2, are not able to alter the equilibrium in unemployment rate. This again contradicts the new Keynesian view of the monetary policy being able to alter the equilibrium in unemployment.

Regarding the response of M2, the expansionary monetary policy, executed by increasing the stock of M2,

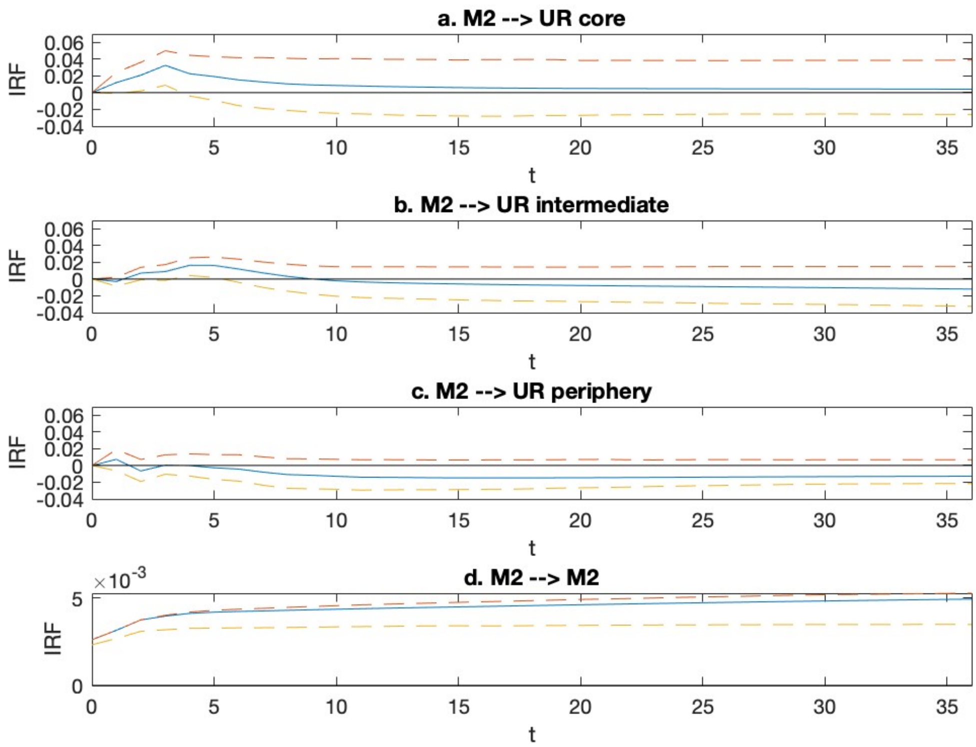

Figure 3.4: impulse response functions of unemployment rate for the core (panel a.), intermediate (panel b.), and peripheral (panel c.) aggregated economies, and of the stock of M2 (panel d.), to a one standard deviation innovation in the stock of M2 of the ECB. The "t" variable represents the months, and the zero indicates the moment in which the ECB adopts its unexpected expansionary monetary policy, i.e. the moment in which it unexpectedly decides to raise the stock of M2. The dotted lines represent the upper and lower bounds of the 68% bootstrap confidence intervals for the estimated impulse responses. The blue solid lines represent the expected value of the impulse response functions. The impulse responses for the remaining variables of the system can be found in Figure 3.3.

necessitates a higher period-by-period growth rate of M2 compared to the growth rate before the innovation. Following the ECB's action and a three-month adaptation period, the impulse response continues to exhibit growth. Figure 3.4d suggests that the ECB tends to sustain a relatively high growth rate for M2 during the initial months following their unexpected action, and this higher growth rate persists in comparison to the pre-innovation growth rate of M2. This indicates that the change in M2 appears to have persistent, if not permanent, effects on the variable itself.

Another noteworthy aspect of this analysis is the observation that the response of all variables to an unexpected innovation in M2 unfolds more rapidly than what was estimated for the interest rate case. Most of the responses and stabilization occur within the first year, as opposed to the two years of the policy rate analysis. Specifically, the decline in industrial production only lasts around two or three months for the core and intermediate economies, instead of the seven and twelve months estimated in the interest rate scenario (Section 3.1).

3.3 M3 Impulse Responses

In this last analysis, the variable used as monetary policy instrument is the stock of M3[3], which again serves as an indicator of unconventional monetary policy measures. The variables of this system are thus

$$y_t = \begin{bmatrix} IP_t^C & IP_t^I & IP_t^P & UR_t^C & UR_t^I & UR_t^P & M3_t \end{bmatrix}'.$$

The Johansen cointegration test suggests a cointegration rank of three ($r = 3$). This implies fewer equilibrium conditions compared to the two previous analyses, although still relatively close. The Akaike information criteria recommend an optimal lag of three ($p = 3$), while the Bayesian information criteria propose a single optimal lag ($p = 1$) for the corresponding VAR(p) model. For the same reasons explained in section 3.2, I opt for the Akaike information criteria suggestion, estimating a VECM(2) with a cointegration rank of three. The details on the estimated parameters of the model, as well as a check on the assumption of Gaussianity and independence of the residuals can be found in Appendix III. In this case as well, due to the low variance of the M3 variable, the asymptotic confidence intervals tend to appear flattened around the expected value. Therefore, I construct the 68% confidence intervals using the bootstrap method.

The estimates from this analysis align with the findings of the previous two analyses and are notably similar to those of M2. The main distinction between the monetary aggregates M2 and M3 lies in the composition of M3, which includes repurchase agreements[4], money market fund shares[5], and debt securities with a maturity of up to two years. Given the roles of these three components in the economy, it is evident that the ECB's objective in altering its stock of M3 is different from that of stimulating economic growth. The increase in M3 primarily provides stability during times of crisis for various types of institutions, including banks, governments, and businesses. Thus, I anticipate that interventions through M3 are primarily aimed at sustaining the economy rather than stimulating it[6].

Regarding industrial production, the core economies (Austria, Belgium, Germany, and the Netherlands) initially experience a decrease before rebounding within a few months and stabilizing at zero (Figure 3.5a), which is the same level as before the innovation. In contrast, industrial production in the peripheral economies (Finland and Ireland) increases and stabilizes at statistically significant levels above zero (Figure 3.5c). Of

[3]M3 is defined as the sum of currency in circulation, overnight deposits, deposits with an agreed maturity of up to two years, deposits redeemable at notice of up to three months, repurchase agreements, money market fund shares/units, and debt securities with a maturity of up to two years.

[4]The repurchase agreement is defined by the ECB as a special form of deposit, as it simultaneously constitutes both a deposit and a collateralised loan. It is, therefore, a twofold transaction, comprising the sale of a security (or a basket of securities) by one party to another party for cash on the initiation date, with the added condition that these assets will be repurchased at a later date established in advance. The main advantage is the high level of security that it offers to the lender in the event of the borrower defaulting, as the risk is covered by the collateral received.

[5]A money market fund is a type of mutual fund. They issue shares to investors to finance their activities, offering a high degree of liquidity, diversification and market-based yields. The value of their shares fluctuates in line with the price of the debt instruments in which they have invested. They have to maintain a high level of asset liquidity to be able to meet daily redemptions by their investors. Money market funds are considered to be systemically relevant entities, due to their close ties to the banking sector and other financial activities. In addition, money market funds are an important source of corporate and government financing, therefore runs on funds (i.e. massive withdrawals from Money market funds) in a financial crisis may have important macroeconomic consequences.

[6]Similarly to M2, M3 is employed with the aim of achieving stabilization objectives, particularly within the context of unconventional monetary policies, as emphasized by Burriel and Galesi (2018). However, it is important to note that M3 serves an even more pronounced role in this regard, given that it includes repurchase agreements, money market fund shares/units, and debt securities with a maturity of up to two years, which directly support both the private and public sectors.

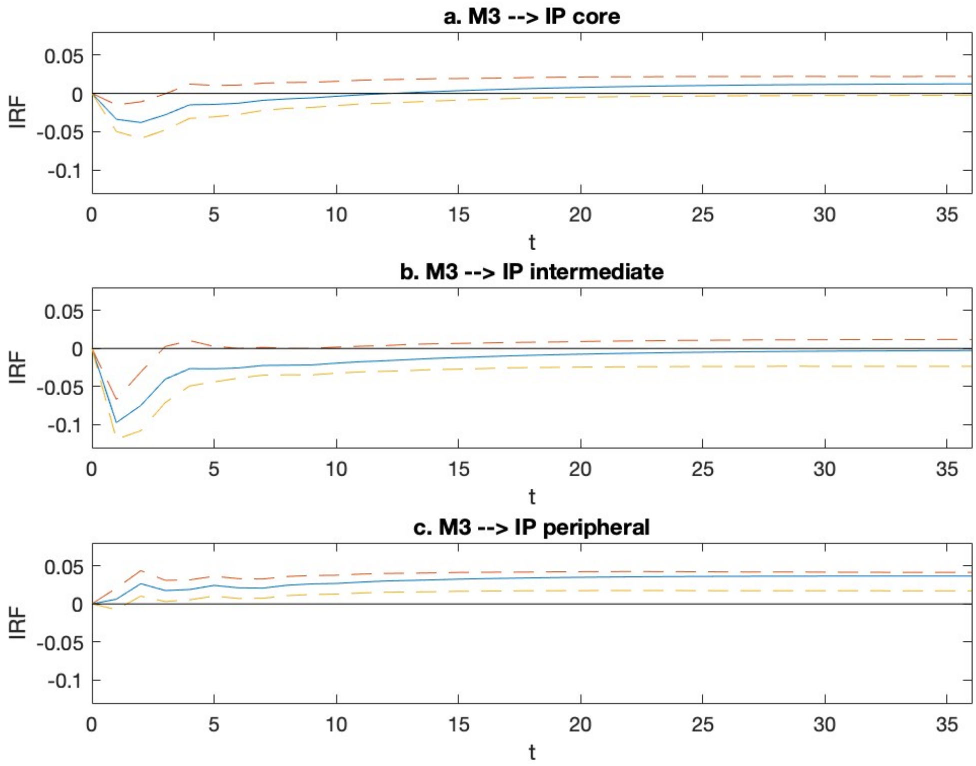

Figure 3.5: impulse response functions of industrial production for the core (panel a.), intermediate (panel b.), and peripheral (panel c.) aggregated economies, to a one standard deviation innovation in the stock of M3 of the ECB. The "t" variable represents the months, and the zero indicates the moment in which the ECB adopts its unexpected expansionary monetary policy, i.e. the moment in which it unexpectedly decides to raise the stock of M3. The dotted lines represent the upper and lower bounds of the 68% bootstrap confidence intervals for the estimated impulse responses. The blue solid lines represent the expected value of the impulse response functions. The impulse responses for the remaining variables of the system can be found in Figure 3.6.

greater concern in terms of impulse responses is the reaction of the intermediate economies (France, Spain, and Italy). These economies (Figure 3.5b) witness a decline in industrial production after the innovation, with a more substantial decrease compared to the core economies, and they stabilize at zero within a few months. This implies that when the ECB decides to support the Euro area economies by acting on the components of M3, it needs to consider that the core and intermediate economies will not be stimulated, whereas the peripheral economies will benefit.

Regarding the unemployment rate variable, the impulse responses, as illustrated in Figure 3.6, exhibit a consistent pattern across the three macro-areas. This pattern aligns with the findings from Section 3.2 for the core and periphery. In these two macro-areas, the unemployment rate remains largely unaffected by the change in monetary policy. However, the unemployment rate in the intermediate economies displays a different behavior, being notably influenced and ultimately stabilizing at levels below the initial rates (Figure 3.6c).

Finally, the impulse response of M3 (Figure 3.5d) reveals the ECB's tendency to respond following an innovation in this variable. The ECB continues to increase M3 after the innovation, suggesting a permanent increase in the variable's growth rate due to the innovation.

It's important to interpret the results of this section in the context of the Euro area potentially facing a crisis,

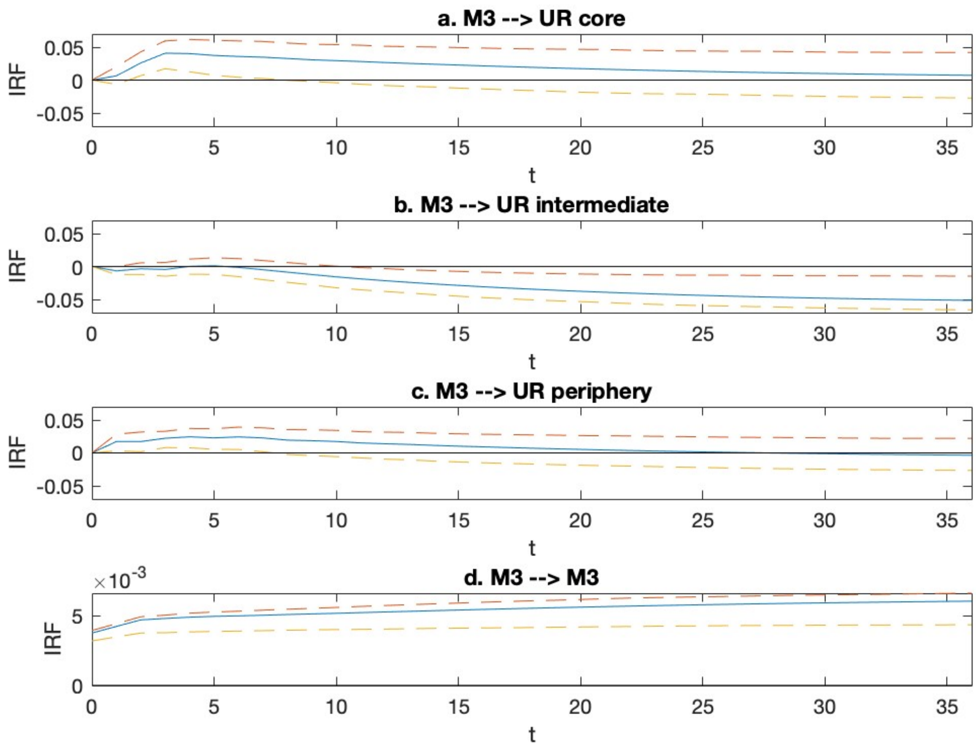

Figure 3.6: impulse response functions of unemployment rate for the core (panel a.), intermediate (panel b.), and peripheral (panel c.) aggregated economies, and of the stock of M3 (panel d.), to a one standard deviation innovation in the stock of M3 of the ECB. The "t" variable represents the months, and the zero indicates the moment in which the ECB adopts its unexpected expansionary monetary policy, i.e. the moment in which it unexpectedly decides to raise the stock of M3. The dotted lines represent the upper and lower bounds of the 68% bootstrap confidence intervals for the estimated impulse responses. The blue solid lines represent the expected value of the impulse response functions. The impulse responses for the remaining variables of the system can be found in Figure 3.5.

rather than the ECB's use of M3 expansion as a tool for economic stimulation. If the proposed interpretation of M3 usage is accurate, where M3 serves primarily as a means to maintain stability during crises rather than stimulate economies, then the figures presented here should be viewed cautiously within this analysis. Nonetheless, the evidence still indicates heterogeneity among Euro area member states in the responses to an increase in M3. Therefore, under the assumption that the ECB is increasing the M3 supply to support the Euro area economy during a crisis, it appears that the core and peripheral economies are those who suffer in terms of unchanged unemployment rate and production, if compared to the peripheral economies. Thus, the core and intermediate economies risk to lag behind with respect to the peripheral economies.

Conclusion

The primary objective of this analysis was to estimate the impact of the ECB's unexpected monetary policy on the national economies within the Euro area. Furthermore, it aimed to assess whether there were variations in responses among the member states. The analysis employed the VECM parameterization to derive impulse response functions for the model's variables in response to shocks in the monetary policy instruments. Three distinct models were estimated, considering three monetary policy variables: the interest rate, the stock of monetary aggregate M2, and the stock of monetary aggregate M3.

The estimated impulse response functions reveal that the interest rate had no significant impact on output through the employment transmission channel. Conversely, unconventional monetary policy instruments exhibited a similar effect on output, in line with theoretical expectations. Expansionary unconventional monetary policies were found to stimulate production, although with variations observed among member states. However, concerns arose regarding the heterogeneity of responses across national economies. Notably, intermediate member states such as France, Spain, and Italy displayed limited responses to expansionary policies, irrespective of the monetary policy measure considered. In contrast, Finland and Ireland experienced a decline in output following an expansionary interest rate policy. There is less concern about the core economies failing to respond to an increase in the M3 stock, as such actions are primarily aimed at stabilizing rather than stimulating the economy.

The impulse responses of interest rates and monetary aggregates (M2 and M3) also provide insights into the ECB's post-innovation monetary policy actions. After interest rate changes, the ECB usually maintained the policy stance by keeping the policy rate below the pre-innovation level, close to the newly set rate. On the other hand, with regard to monetary aggregates M2 and M3, the ECB tended to sustain higher growth rates following the innovation, resulting in a permanent influence on the trend of these variables.

In summary, this analysis highlights the existence of heterogeneity in the reactions of Euro area member states to the ECB's monetary policy interventions. Specifically, France, Spain, and Italy exhibited unresponsive behaviors, setting them apart from the other Euro area economies under consideration. Consequently, there is a risk that the ECB's implementation of expansionary monetary policies may not equally benefit these economies in terms of stimulus. Furthermore, this heterogeneity contributes to an increase in the "costs" that Euro area member states must contend with as participants in the monetary union.

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Appendix

Appendix I: Details of Interest Rate's VECM Estimation

The Akaike Information Criteria (AIC) and Bayesian Information Criteria (BIC) for the VECM model in Section 3.1 are found in Table A.1. They are derived from the estimated VAR model associated with the VECM, for values of p ranging from one to twelve. In particular, the best-fit model according to AIC and BIC is the one that explains the greatest amount of variation using the fewest possible independent variables. In fact, they adopt two different penalty terms, which vary with the number of p lags employed, on the likelihood of the specific model. In particular, the penalty term of the BIC tends to be larger as the model dimension increases.

Table A.1: this table reports the values for the AIC and the BIC of the VECM($p-1$) model in Section 3.1. The optimal value for p is the one which minimizes the AIC or the BIC.

p	AIC	BIC
1	-128.4591	77.247
2	-285.4166	100.2823
3	-407.6737	158.018
4	-406.2996	339.3851
5	-373.1363	552.5411
6	-344.754	760.9163
7	-309.5389	976.1242
8	-302.6007	1163.0553
9	-279.6017	1366.0471
10	-247.5817	1578.06
11	-231.1152	1774.5193
12	-211.1894	1974.438

The optimal value of p under the AIC and BIC is the one that minimizes the AIC or BIC value. The AIC, which is the selection method employed in all three analyses, suggests an optimal lag of $p = 3$. Therefore, the VECM model for Section 3.1 is a VECM(2)

$$\Delta y_t = \alpha\beta' y_{t-1} + \Gamma_1 \Delta y_{t-1} + \Gamma_2 \Delta y_{t-2} + u_t \quad . \tag{3.1}$$

The cointegration rank of α and β is $r = 2$ as provided by the Johansen cointegration test. To be more precise in the specification, intercepts are present in the cointegrating relations, and deterministic linear trends are present in the levels of the data, so that the cointegration term is actually

$\alpha(\beta y_{t-1} + c_0) + c_1$

The "estimate" function of Matlab for VECM models allows to estimate these parameters by maximum likelihood.

The ML estimate for $\Pi = \alpha\beta'$ is reported in Table A.2, while those for the Γ_1 and Γ_2 matrices are found in Table A.3 and Table A.4.

Table A.2: estimated Π parameter, as derived in Section 2.4. Note that the rank of the α and β matrices is r = 2, i.e. the cointegration rank is two. The estimated values are displayed together with their p-value, which is reported in brackets below the element to which it corresponds.

			$\tilde{\Pi}$			
-0.094	0.006	0.102	0.002	0.051	-0.021	-0.025
(0.001)	(0.838)	(0.032)	(0.762)	(0.024)	(0.302)	(0.044)
-0.152	-0.082	0.073	0.029	0.042	-0.112	-0.015
(0)	(0.045)	(0.315)	(0.007)	(0.216)	(0)	(0.434)
0.030	-0.071	-0.103	0.018	-0.047	-0.051	0.028
(0.218)	(0.001)	(0.010)	(0.002)	(0.013)	(0.003)	(0.008)
0.006	-0.065	-0.072	0.017	-0.032	-0.053	0.020
(0.796)	(0.002)	(0.055)	(0.001)	(0.073)	(0.001)	(0.042)
0.055	-0.051	-0.108	0.011	-0.051	-0.027	0.028
(0)	(0)	(0)	(0)	(0.001)	(0.001)	(0)
0.016	-0.096	-0.114	0.026	-0.051	-0.076	0.031
(0.473)	(0)	(0.001)	(0)	(0)	(0)	(0.001)
-0.015	0.016	0.032	-0.004	0.015	0.009	-0.008
(0.273)	(0.191)	(0.151)	(0.230)	(0.155)	(0.318)	(0.149)

Table A.3: estimated Γ_1 parameter, as derived in Section 2.4. The estimated values are displayed together with their p-value, which is reported in brackets below the element to which it corresponds.

			$\tilde{\Gamma}_1$			
-0.418	0.269	-0.069	0.021	0.221	-0.053	0.223
(0)	(0)	(0.001)	(0.929)	(0.229)	(0.358)	(0.061)
-0.294	0.323	-0.059	-0.065	0.983	-0.080	0.352
(0.182)	(0)	(0.038)	(0.429)	(0)	(0.333)	(0.045)
0.172	0.019	-0.350	-0.108	-0.040	-0.051	0.328
(0.015)	(0.028)	(0)	(0.099)	(0.064)	(0.272)	(0.010)
-0.029	0.012	0.040	-0.160	0.124	0.108	-0.120
(0.733)	(0.917)	(0.320)	(0.010)	(0.212)	(0.057)	(0.162)
-0.039	0.087	-0.031	0.070	0.087	0.022	-0.019
(0.012)	(0)	(0)	(0.005)	(0.002)	(0.313)	(0.583)
-0.037	0	0.100	0.096	0.444	-0.526	-0.023
(0.614)	(0.479)	(0.007)	(0.111)	(0)	(0)	(0.668)
0.025	0.010	-0.030	0.025	-0.089	0.037	0.176
(0.378)	(0.889)	(0.230)	(0.525)	(0.331)	(0.221)	(0.001)

Table A.4: estimated Γ_2 parameter, as derived in Section 2.4. The estimated values are displayed together with their p-value, which is reported in brackets below the element to which it corresponds.

$\tilde{\Gamma}_2$

-0.138	0	0.122	-0.014	0.038	-0.047	0.225
(0.250)	(0.891)	(0.067)	(0.504)	(0.000)	(0.009)	(0.679)
0.007	-0.238	0.267	-0.052	-0.413	-0.054	0.455
(0.001)	(0.753)	(0.170)	(0.798)	(0.972)	(0.004)	(0.003)
0.162	-0.039	-0.296	-0.037	-0.276	0.039	0.156
(0.635)	(0.135)	(0.006)	(0.000)	(0.000)	(0.000)	(0.000)
0.012	-0.093	0.088	0.029	-0.082	0.130	-0.007
(0.000)	(0.568)	(0.524)	(0.635)	(0.135)	(0.006)	(0.479)
-0.058	0.087	-0.031	0.053	0.067	0.006	-0.058
(0.000)	(0.888)	(0)	(0.000)	(0.568)	(0.524)	(0.635)
-0.047	0.035	0.125	0.045	0.188	-0.376	-0.016
(0.000)	(0.353)	(0.831)	(0.000)	(0.000)	(0.000)	(0.000)
0.002	0.005	-0.014	0.008	-0.125	0.013	0.332
(0.000)	(0.000)	(0.770)	(0.476)	(0.353)	(0.830)	(0)

In Section 2.4, the Gaussianity of the error terms, and thus of the process of y_t was assumed. More precisely, the error terms were assumed to be independent and identically distributed (iid) as $u_t \sim N(0, \Sigma_u)$. In this appendix I check the validity of such assumption. The formal Ljung-Box Q-test, which assesses the hypothesis of zero autocorrelation (which implies independence) among the residuals versus the alternative of the presence of at least one non-zero autocorrelation, is known to be quite stringent. With a limited sample size, achieving zero autocorrelation based on this test can be challenging. As such, the assumption of no autocorrelation in the residuals and of their normality is informally examined. First I provide the sample autocorrelation (Figure A.1 and Figure A.2) and partial autocorrelation (Figure A.3 and Figure A.4) functions. Consequently, I provide the sample distribution of the residuals (Figure A.5 and Figure A.6).

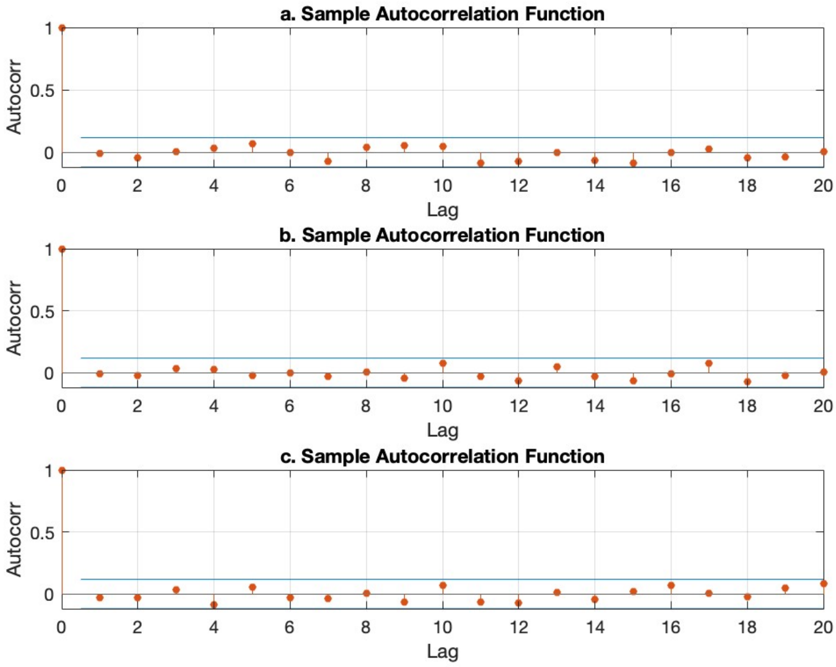

Figure A.1: this figure reports the autocorrelation functions for the residuals of the first (a.), the second (b.), and the third (c.) variable of the system. Notably, there is no autocorrelation in the residuals considered, as all the autocorrelations fall inside the 95% confidence interval.

Figure A.1, Figure A.2, Figure A.3, and Figure A.4 indicate that the residuals exhibit no signs of autocorrelation nor partial autocorrelation, thereby confirming the assumption of no autocorrelation among the residuals. This implies that the residuals are independent within themselves. The next step is to confirm their normal distribution, ensuring that the assumptions made during model estimation regarding the error term distribution hold valid. This validation justifies the use of the ML estimators, as derived in Section 2.4.

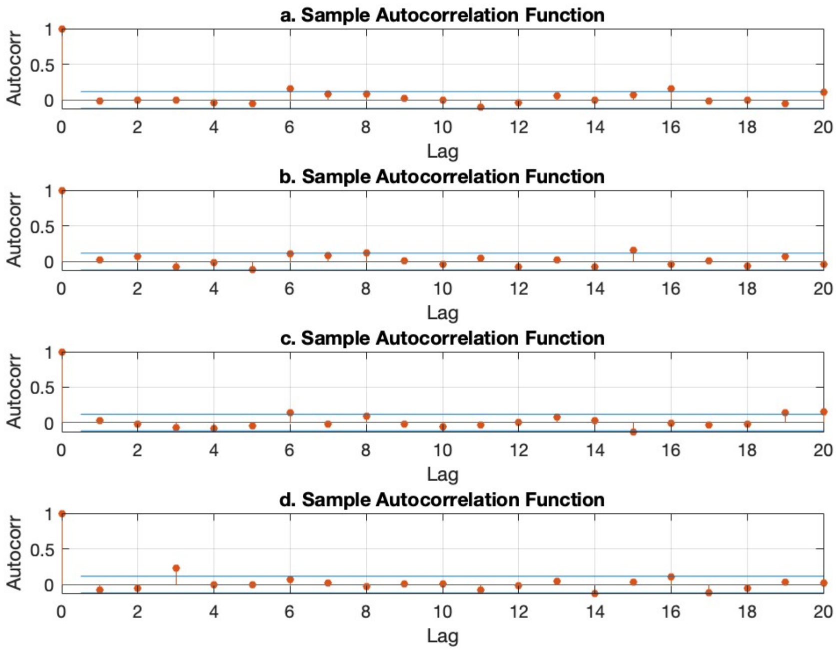

Figure A.2: this figure reports the autocorrelation functions for the residuals of the fourth (a.), the fifth (b.), the sixth (c.), and the seventh (d.) variable of the system. Notably, there is no autocorrelation in the residuals considered, as all the autocorrelations fall inside the 95% confidence interval.

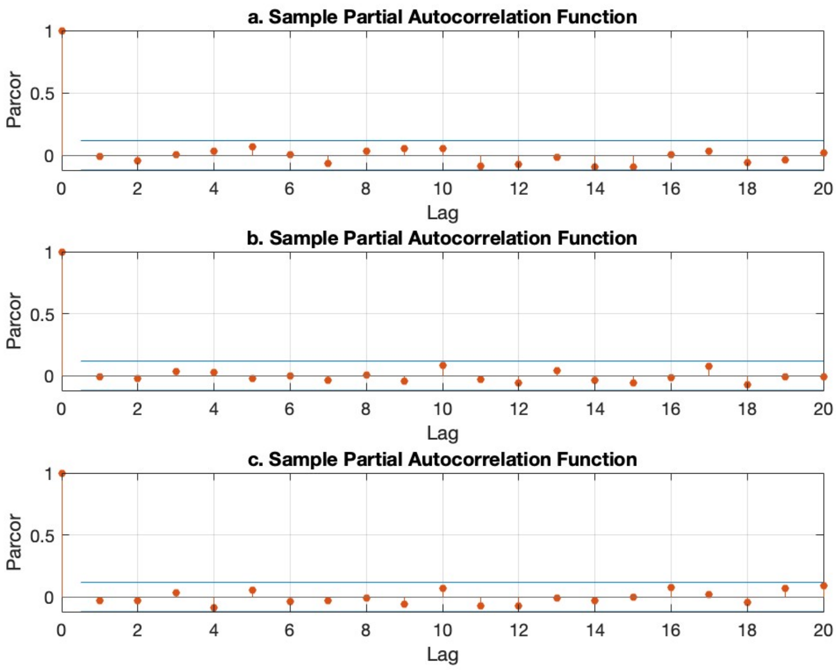

Figure A.3: this figure reports the partial autocorrelation functions for the residuals of the first (a.), the second (b.), and the third (c.) variable of the system. Notably, there is no partial autocorrelation in the residuals considered, as all the partial autocorrelations fall inside the 95% confidence interval.

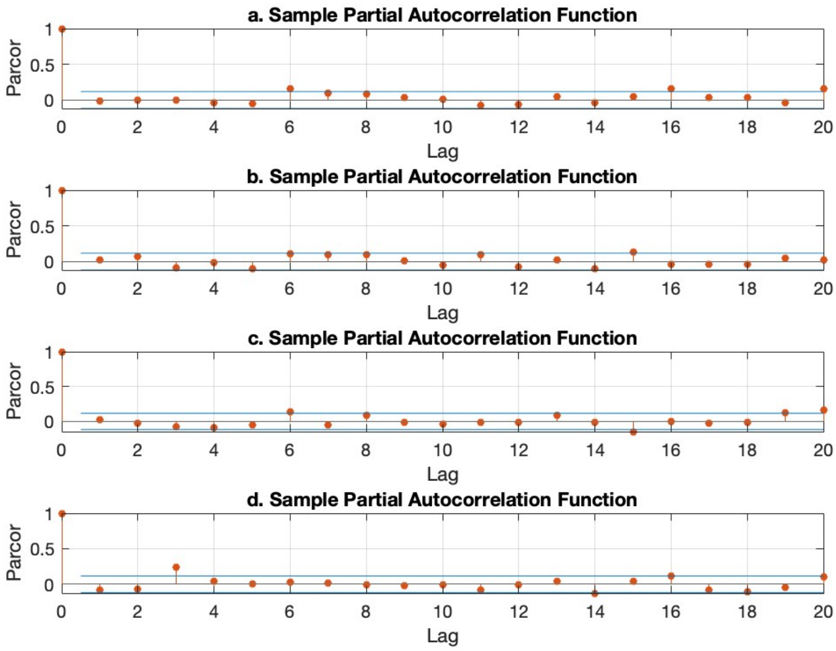

Figure A.4: this figure reports the partial autocorrelation functions for the residuals of the fourth (a.), the fifth (b.), the sixth (c.), and the seventh (d.) variable of the system. Notably, there is no partial autocorrelation in the residuals considered, as all the partial autocorrelations fall inside the 95% confidence interval.

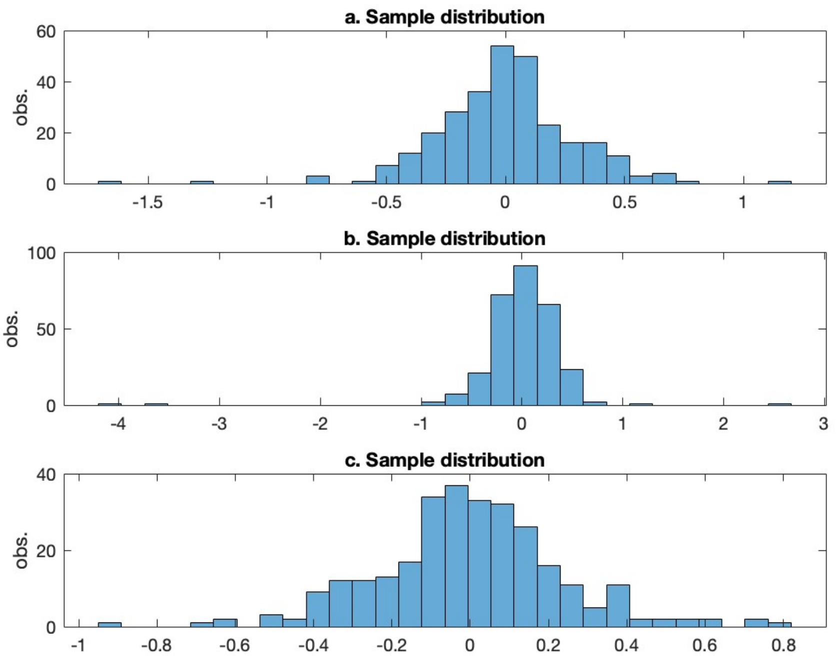

Figure A.5: this figure reports the sample distribution of the residuals of the first (a.), the second (b.), and the third (c.) variable of the system.

Notably, both Figure A.5 and Figure A.6 depict residuals that appear to be normally distributed or very close to it. Given the relatively small sample size, which may not fully capture the population distribution, these observations provide justification for assuming normality within the context of this analysis.

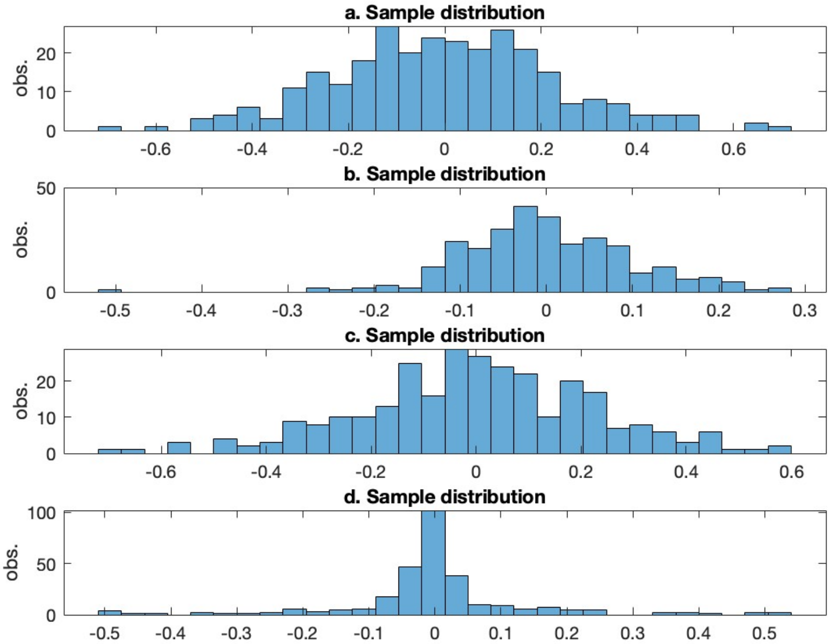

Figure A.6: this figure reports the sample distribution of the residuals of the fourth (a.), the fifth (b.), the sixth (c.), and the seventh (d.) variable of the system.

The informal residuals diagnostic conducted shows that the residuals are both independent with respect to themselves, and are normally distributed with zero mean. This validates the use of ML to estimate the parameters under normality of the error terms.

Appendix II: Details of M2's VECM Estimation

The Akaike Information Criteria (AIC) and Bayesian Information Criteria (BIC) for the VECM model in Section 3.2 are found in Table A.5.

Table A.5: this table reports the values for the AIC and the BIC of the VECM($p-1$) model in Section 3.2. The optimal value for p is the one which minimizes the AIC or the BIC.

p	AIC	BIC
1	-2455.9534	-2250.2473
2	-2618.8466	-2233.1476
3	-2723.5832	-2157.8914
4	-2682.953	-1937.2684
5	-2654.4741	-1728.7966
6	-2607.5759	-1501.9056
7	-2586.9267	-1301.2636
8	-2575.4829	-1109.8269
9	-2550.8536	-905.2047
10	-2499.5225	-673.8808
11	-2452.6633	-447.0288
12	-2422.9994	-237.3721

The optimal value of p under the AIC and BIC is the one that minimizes the AIC or BIC value. The AIC, which is the selection method employed in all three analyses, suggests an optimal lag of $p = 3$. Therefore, the

VECM model for Section 3.2 is a VECM(2)

$$\Delta y_t = \alpha\beta' y_{t-1} + \Gamma_1 \Delta y_{t-1} + \Gamma_2 \Delta y_{t-2} + u_t \quad . \tag{3.2}$$

The cointegration rank of α and β is $r = 4$ as provided by the Johansen cointegration test. To be more precise in the specification, intercepts are present in the cointegrating relations, and deterministic linear trends are present in the levels of the data, so that the cointegration term is actually

$$\alpha(\beta y_{t-1} + c_0) + c_1$$

The "estimate" function of Matlab for VECM models allows to estimate these parameters by maximum likelihood.

The ML estimate for $\Pi = \alpha\beta'$ is reported in Table A.6, while those for the Γ_1 and Γ_2 matrices are found in Table A.7 and Table A.8.

Table A.6: estimated Π parameter, as derived in Section 2.4. Note that the rank of the α and β matrices is $r = 2$, i.e. the cointegration rank is two. The estimated values are displayed together with their p-value, which is reported in brackets below the element to which it corresponds.

$\tilde{\Pi}$

-0.14047 (0.000)	0.082204 (0.082)	0.000409 (0.000)	0.001517 (0.002)	0.037398 (0.097)	0.097878 (0.127)	0.127302 (0.020)
-0.13234 (0.009)	-0.18686 (0.000)	0.35687 (0.065)	0.016949 (0.107)	0.064273 (0.000)	-0.01839 (0.001)	-1.0253 (0.066)
0.018856 (0.440)	0.061357 (0.001)	-0.32994 (0.000)	0.005282 (0.002)	-0.075682 (0.001)	-0.04001 (0.066)	0.98044 (0.107)
0.048727 (0.001)	-0.12333 (0.595)	-0.032404 (0.718)	0.011669 (0.238)	-0.054611 (0.631)	-0.040157 (0.226)	-0.52548 (0.131)
0.095876 (0.008)	-0.12516 (0.000)	0.009419 (0.065)	0.004654 (0.107)	-0.055031 (0.000)	-0.012271 (0.038)	-0.81518 (0.051)
0.030971 (0.001)	-0.11819 (0.065)	-0.070051 (0.002)	0.02821 (0.000)	-0.032794 (0.001)	-0.14473 (0.066)	-0.42254 (0.127)
0.000392 (0.309)	-0.000434 (0.000)	-0.000348 (0.000)	0.000265 (0.040)	0.000159 (0.051)	-0.001715 (0.001)	-0.003446 (0.066)

Table A.7: estimated Γ_1 parameter, as derived in Section 2.4. The estimated values are displayed together with their p-value, which is reported in brackets below the element to which it corresponds.

$\tilde{\Gamma}_1$

-0.286324 (0.000)	0.222467 (0.913)	-0.129327 (0.461)	0.0390895 (0.837)	0.129671 (0.002)	-0.139128 (0.038)	-16.0255 (0.079)
-0.0129693 (0.109)	0.283968 (0.164)	-0.291819 (0.000)	-0.116312 (0.038)	0.807101 (0.430)	-0.218467 (0.100)	-39.3059 (0.018)
0.16331 (0.002)	0.033304 (0.405)	-0.23574 (0.305)	-0.053499 (0.770)	-0.19842 (0.080)	-0.06567 (0.930)	-7.1768 (0.297)
-0.055284 (0.645)	0.034233 (0.734)	0.013269 (0.001)	-0.18395 (0.429)	0.14496 (0.110)	0.09 (0.000)	5.1357 (0.500)
-0.075129 (0.001)	0.096553 (0.406)	0.0032791 (0.310)	0.054317 (0.769)	0.12496 (0.768)	0.0069895 (0.145)	-0.29408 (0.000)
-0.037426 (0.000)	-0.018076 (0.001)	0.10321 (0.913)	0.094132 (0.430)	0.54394 (0.100)	-0.48219 (0.019)	3.2285 (0.046)
0.0013526 (0.000)	-0.00035533 (0.002)	0.0009112 (0.003)	0.00054118 (0.000)	-0.0034378 (0.002)	0.0013993 (0.000)	0.21013 (0.500)

Table A.8: estimated Γ_2 parameter, as derived in Section 2.4. The estimated values are displayed together with their p-value, which is reported in brackets below the element to which it corresponds.

$\tilde{\Gamma}_2$

-0.0728002 (0.363)	-0.0319381 (0.165)	0.0851707 (0.015)	0.0243166 (0.912)	-0.0349737 (0.000)	-0.0893356 (0.573)	21.0891 (0.507)
0.163223 (0.560)	-0.252367 (0.001)	0.123563 (0.000)	-0.0480348 (0.914)	-0.420915 (0.005)	-0.138646 (0.553)	42.3119 (0.080)
0.1625 (0.004)	-0.043097 (0.365)	-0.20312 (0.110)	0.032356 (0.001)	-0.42946 (0.001)	0.049343 (0.837)	7.8665 (0.001)
-0.0070428 (0.125)	-0.070094 (0.040)	0.066256 (0.115)	0.0040607 (0.774)	-0.040237 (0.261)	0.11191 (0.238)	-0.11689 (0.170)
-0.090856 (0.001)	0.099761 (0.015)	-0.018165 (0.002)	0.036274 (0.913)	0.10913 (0.007)	0.0022829 (0.000)	3.9621 (0.165)
-0.034987 (0.000)	0.013476 (0.925)	0.12674 (0.997)	0.034433 (0.100)	0.24697 (0.235)	-0.34861 (0.630)	-9.1085 (0.305)
0.00050269 (0.057)	-1.5864e-05 (0.054)	-0.00020742 (0.001)	0.00016177 (0.836)	-0.00029523 (0.001)	0.0016246 (0.000)	0.17924 (0.549)

In Section 2.4, the Gaussianity of the error terms, and thus of the process of y_t was assumed. More precisely, the error terms were assumed to be independent and identically distributed (iid) as $u_t \sim N(0, \Sigma_u)$. In this appendix I check the validity of such assumption. The formal Ljung-Box Q-test, which assesses the hypothesis of zero autocorrelation (which implies independence) among the residuals versus the alternative of the presence of at least one non-zero autocorrelation, is known to be quite stringent. With a limited sample size, achieving zero autocorrelation based on this test can be challenging. As such, the assumption of no autocorrelation in the residuals and of their normality is informally examined. First I provide the sample autocorrelation (Figure A.7 and Figure A.8) and partial autocorrelation (Figure A.9 and Figure A.10) functions. Consequently, I provide the sample distribution of the residuals (Figure A.11 and Figure A.12).

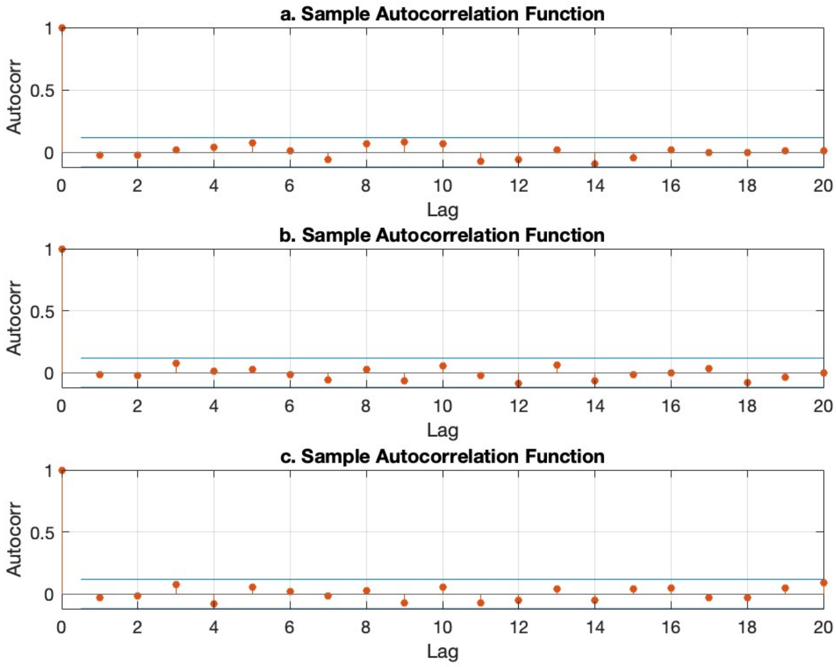

Figure A.7: this figure reports the autocorrelation functions for the residuals of the first (a.), the second (b.), and the third (c.) variable of the system. Notably, there is no autocorrelation in the residuals considered, as all the autocorrelations fall inside the 95% confidence interval.

Figure A.7, Figure A.8, Figure A.9, and Figure A.10 indicate that the residuals exhibit no signs of autocorrelation nor partial autocorrelation, thereby confirming the assumption of no autocorrelation among the residuals. This implies that the residuals are independent within themselves. The next step is to confirm their normal distribution, ensuring that the assumptions made during model estimation regarding the error term distribution hold valid. This validation justifies the use of the ML estimators, as derived in Section 2.4.

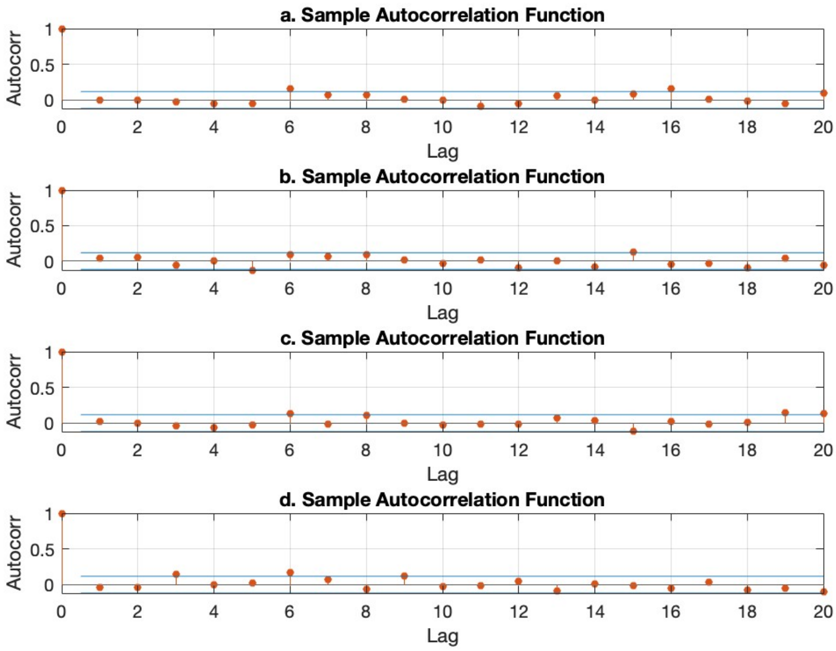

Figure A.8: this figure reports the autocorrelation functions for the residuals of the fourth (a.), the fifth (b.), the sixth (c.), and the seventh (d.) variable of the system. Notably, there is no autocorrelation in the residuals considered, as all the autocorrelations fall inside the 95% confidence interval.

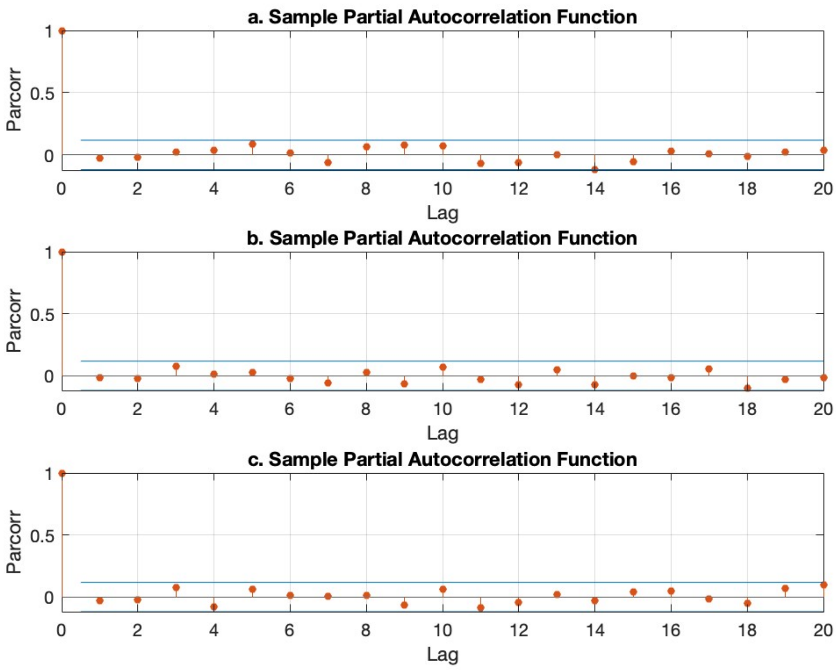

Figure A.9: this figure reports the partial autocorrelation functions for the residuals of the first (a.), the second (b.), and the third (c.) variable of the system. Notably, there is no partial autocorrelation in the residuals considered, as all the partial autocorrelations fall inside the 95% confidence interval.

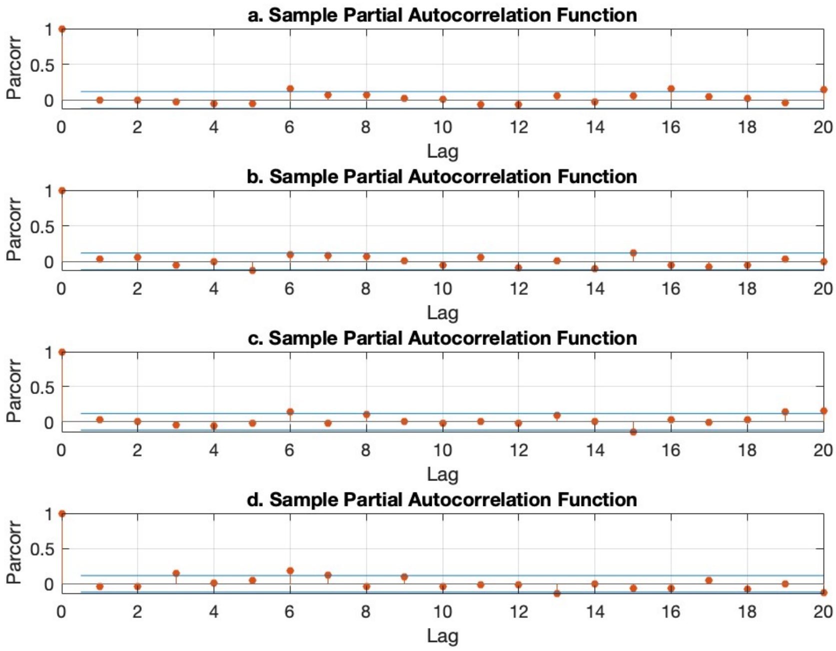

Figure A.10: this figure reports the partial autocorrelation functions for the residuals of the fourth (a.), the fifth (b.), the sixth (c.), and the seventh (d.) variable of the system. Notably, there is no partial autocorrelation in the residuals considered, as all the partial autocorrelations fall inside the 95% confidence interval.

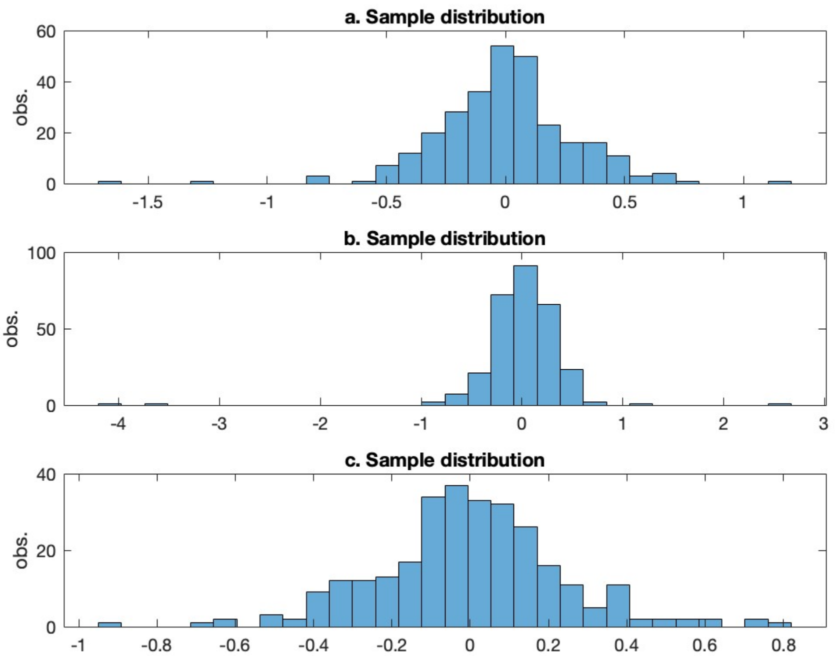

Figure A.11: this figure reports the sample distribution of the residuals of the first (a.), the second (b.), and the third (c.) variable of the system.

Notably, both Figure A.11 and Figure A.12 depict residuals that appear to be normally distributed or very close to it. Given the relatively small sample size, which may not fully capture the population distribution, these observations provide justification for assuming normality within the context of this analysis.

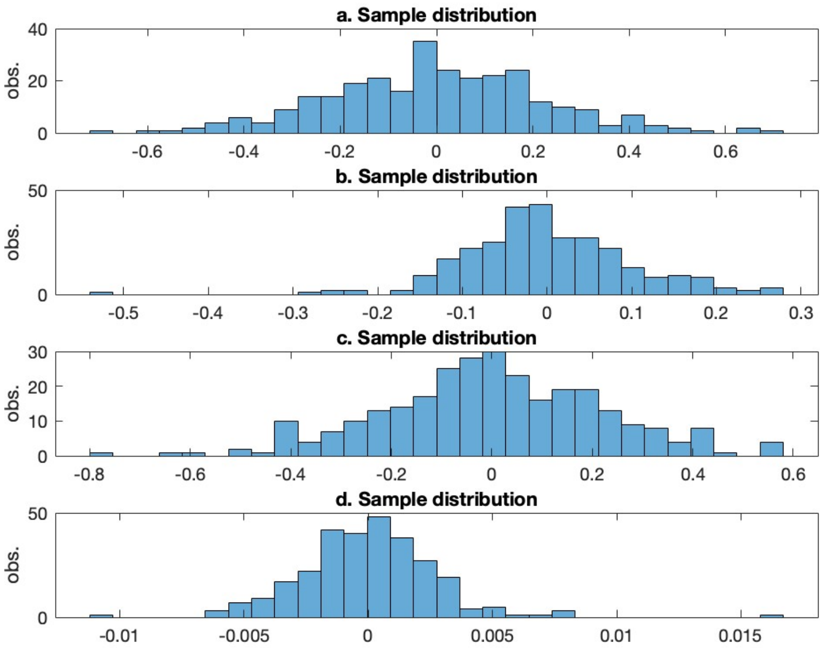

Figure A.12: this figure reports the sample distribution of the residuals of the fourth (a.), the fifth (b.), the sixth (c.), and the seventh (d.) variable of the system.

The informal residuals diagnostic conducted shows that the residuals are both independent with respect to themselves, and are normally distributed with zero mean. This validates the use of ML to estimate the parameters under normality of the error terms.

Appendix III: Details of M3's VECM Estimation

The Akaike Information Criteria (AIC) and Bayesian Information Criteria (BIC) for the VECM model in Section 3.3 are found in Table A.9.

Table A.9: this table reports the values for the AIC and the BIC of the VECM($p-1$) model in Section 3.3. The optimal value for p is the one which minimizes the AIC or the BIC.

p	AIC	BIC
1	-2270.6899	-2064.9838
2	-2415.9081	-2030.2092
3	-2493.2618	-1927.57
4	-2452.3077	-1706.6231
5	-2426.2698	-1500.5924
6	-2385.9419	-1280.2716
7	-2353.812	-1068.1489
8	-2349.0887	-883.4327
9	-2322.0718	-676.423
10	-2276.3535	-450.7118
11	-2234.4262	-228.7917
12	-2199.8943	-14.267

The optimal value of p under the AIC and BIC is the one that minimizes the AIC or BIC value. The AIC, which is the selection method employed in all three analyses, suggests an optimal lag of $p = 3$. Therefore, the

VECM model for Section 3.3 is a VECM(2)

$$\Delta y_t = \alpha\beta' y_{t-1} + \Gamma_1 \Delta y_{t-1} + \Gamma_2 \Delta y_{t-2} + u_t \quad . \tag{3.3}$$

The cointegration rank of α and β is $r = 3$ as provided by the Johansen cointegration test. To be more precise in the specification, intercepts are present in the cointegrating relations, and deterministic linear trends are present in the levels of the data, so that the cointegration term is actually

$$\alpha(\beta y_{t-1} + c_0) + c_1$$

The "estimate" function of Matlab for VECM models allows to estimate these parameters by maximum likelihood.

The ML estimate for $\Pi = \alpha\beta'$ is reported in Table A.10, while those for the Γ_1 and Γ_2 matrices are found in Table A.11 and Table A.12.

Table A.10: estimated Π parameter, as derived in Section 2.4. Note that the rank of the α and β matrices is $r = 2$, i.e. the cointegration rank is two. The estimated values are displayed together with their p-value, which is reported in brackets below the element to which it corresponds.

			$\tilde{\Pi}$			
-0.12371	0.055	0.043	0.000	0.064	-0.015	0.579
(0.002)	(0.005)	(0.005)	(0.000)	(0.000)	(0.000)	(0.889)
-0.087	-0.200	0.210	0.042	0.028	-0.154	-1.042
(0.211)	(0.067)	(0.008)	(0.000)	(0.000)	(0.000)	(0.000)
0.004	0.038	-0.211	-0.007	-0.050	0.032	0.781
(0.001)	(0.000)	(0.015)	(0.000)	(0.000)	(0.002)	(0.746)
0.041	-0.100	-0.063	0.014	-0.058	-0.039	-0.348
(0.000)	(0.000)	(0.000)	(0.043)	(0.015)	(0.002)	(0.000)
0.102	-0.116	-0.029	0.012	-0.071	-0.027	-0.761
(0.002)	(0.000)	(0.001)	(0.003)	(0.000)	(0.000)	(0.001)
0.007	-0.093	-0.016	0.015	-0.033	-0.049	-0.328
(0.000)	(0.190)	(0.043)	(0.008)	(0.004)	(0.014)	(0.000)
-0.000	-0.000	0.003	0.000	0.000	-0.000	-0.013
(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.002)

Table A.11: estimated Γ_1 parameter, as derived in Section 2.4. The estimated values are displayed together with their p-value, which is reported in brackets below the element to which it corresponds.

$\tilde{\Gamma}_1$

-0.305	0.250	-0.147	0.027	0.098	-0.079	-9.542
(0.000)	(0.248)	(0.676)	(0.001)	(0.394)	(0.858)	(0.052)
-0.077	0.316	-0.258	-0.132	0.827	-0.120	-24.739
(0.000)	(0.150)	(0.004)	(0.000)	(0.013)	(0.001)	(0.857)
0.169	0.052	-0.327	-0.054	-0.237	-0.119	0.851
(0.000)	(0.676)	(0.001)	(0.248)	(0.405)	(0.005)	(0.000)
-0.048	0.018	0.038	-0.173	0.176	0.092	2.063
(0.676)	(0.851)	(0.022)	(0.738)	(0.179)	(0.253)	(0.004)
-0.083	0.090	0.026	0.058	0.155	0.022	-0.997
(0.001)	(0.005)	(0.030)	(0.073)	(0.257)	(0.041)	(0.025)
-0.029	-0.036	0.078	0.109	0.569	-0.545	4.912
(0.001)	(0.007)	(0.005)	(0.073)	(0.153)	(0.019)	(0.055)
0.002	-0.000	-0.004	-0.000	-0.004	0.001	0.140
(0.005)	(0.317)	(0.181)	(0.000)	(0.555)	(0.009)	(0.818)

Table A.12: estimated Γ_2 parameter, as derived in Section 2.4. The estimated values are displayed together with their p-value, which is reported in brackets below the element to which it corresponds.

$\tilde{\Gamma}_2$

-0.059	-0.022	0.074	-0.003	-0.049	-0.063	4.309
(0.467)	(0.003)	(0.017)	(0.001)	(0.591)	(0.363)	(0.816)
0.156	-0.245	0.119	-0.105	-0.378	-0.086	14.743
(0.000)	(0.199)	(0.450)	(0.040)	(0.857)	(0.021)	(0.387)
0.161	-0.025	-0.259	0.027	-0.424	0.019	9.184
(0.698)	(0.105)	(0.921)	(0.073)	(0.053)	(0.930)	(0.130)
-0.006	-0.085	0.082	0.015	-0.022	0.118	3.406
(0.166)	(0.241)	(0.724)	(0.078)	(0.978)	(0.042)	(0.130)
-0.092	0.092	-0.007	0.037	0.126	0.013	1.374
(0.166)	(0.006)	(0.514)	(0.244)	(0.130)	(0.323)	(0.398)
-0.048	0.008	0.107	0.053	0.259	-0.377	-0.443
(0.077)	(0.854)	(0.336)	(0.018)	(0.852)	(0.386)	(0.132)
0.001	-0.000	-0.002	-0.000	-0.001	0.001	0.104
(0.131)	(0.131)	(0.888)	(0.386)	(0.275)	(0.103)	(0.174)

In Section 2.4, the Gaussianity of the error terms, and thus of the process of y_t was assumed. More precisely, the error terms were assumed to be independent and identically distributed (iid) as $u_t \sim N(0, \Sigma_u)$. In this appendix I check the validity of such assumption. The formal Ljung-Box Q-test, which assesses the hypothesis of zero autocorrelation (which implies independence) among the residuals versus the alternative of the presence of at least one non-zero autocorrelation, is known to be quite stringent. With a limited sample size, achieving zero autocorrelation based on this test can be challenging. As such, the assumption of no autocorrelation in the residuals and of their normality is informally examined. First I provide the sample autocorrelation (Figure A.13 and Figure A.14) and partial autocorrelation (Figure A.15 and Figure A.16) functions. Consequently, I provide the sample distribution of the residuals (Figure A.17 and Figure A.18).

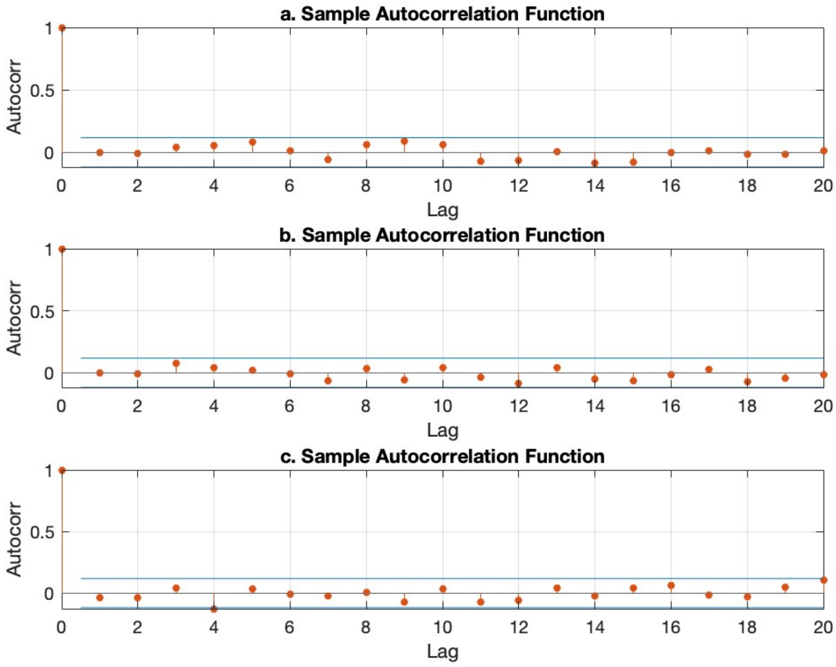

Figure A.13: this figure reports the autocorrelation functions for the residuals of the first (a.), the second (b.), and the third (c.) variable of the system. Notably, there is no autocorrelation in the residuals considered, as all the autocorrelations fall inside the 95% confidence interval.

Figure A.13, Figure A.14, Figure A.15, and Figure A.16 indicate that the residuals exhibit no signs of autocorrelation nor partial autocorrelation, thereby confirming the assumption of no autocorrelation among the residuals. This implies that the residuals are independent within themselves. The next step is to confirm their normal distribution, ensuring that the assumptions made during model estimation regarding the error term distribution hold valid. This validation justifies the use of the ML estimators, as derived in Section 2.4.

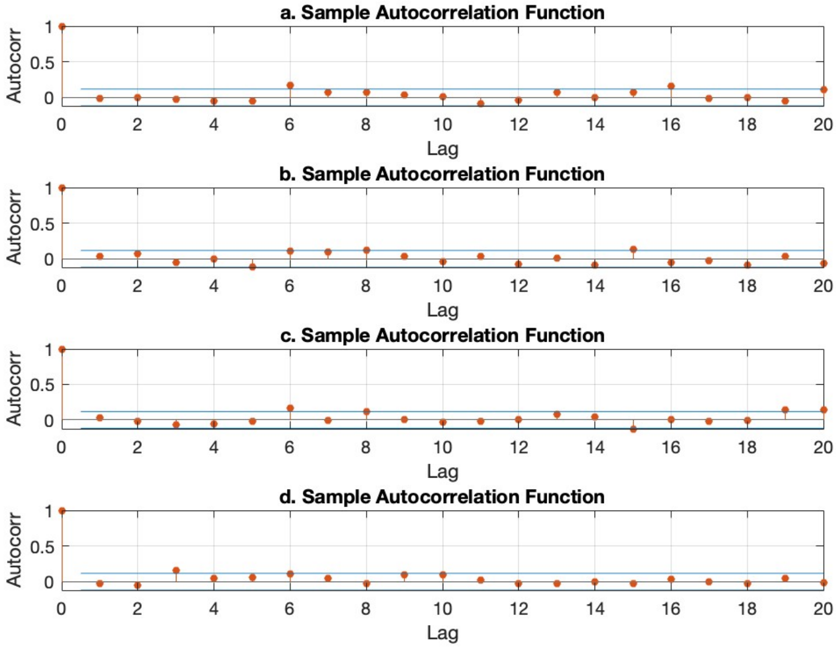

Figure A.14: this figure reports the autocorrelation functions for the residuals of the fourth (a.), the fifth (b.), the sixth (c.), and the seventh (d.) variable of the system. Notably, there is no autocorrelation in the residuals considered, as all the autocorrelations fall inside the 95% confidence interval.

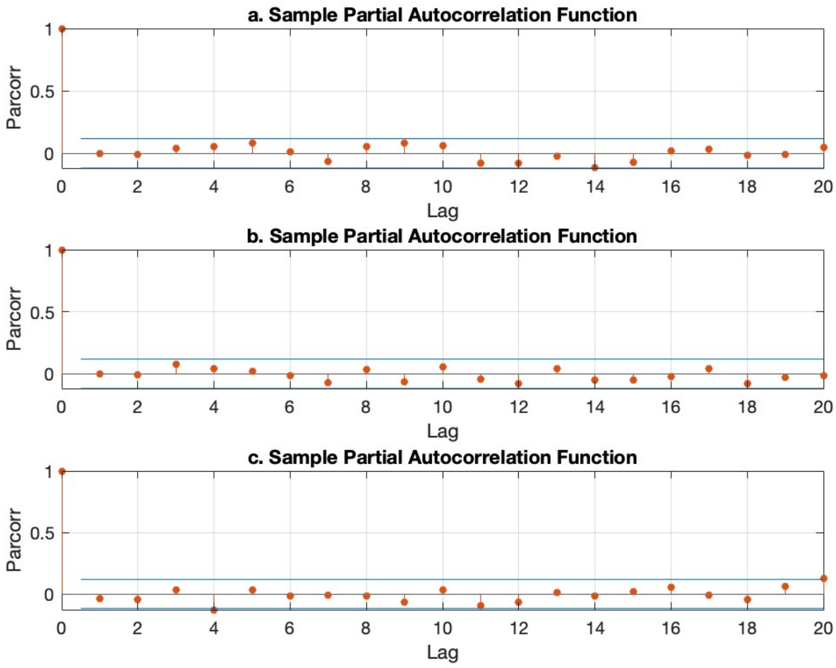

Figure A.15: this figure reports the partial autocorrelation functions for the residuals of the first (a.), the second (b.), and the third (c.) variable of the system. Notably, there is no partial autocorrelation in the residuals considered, as all the partial autocorrelations fall inside the 95% confidence interval.

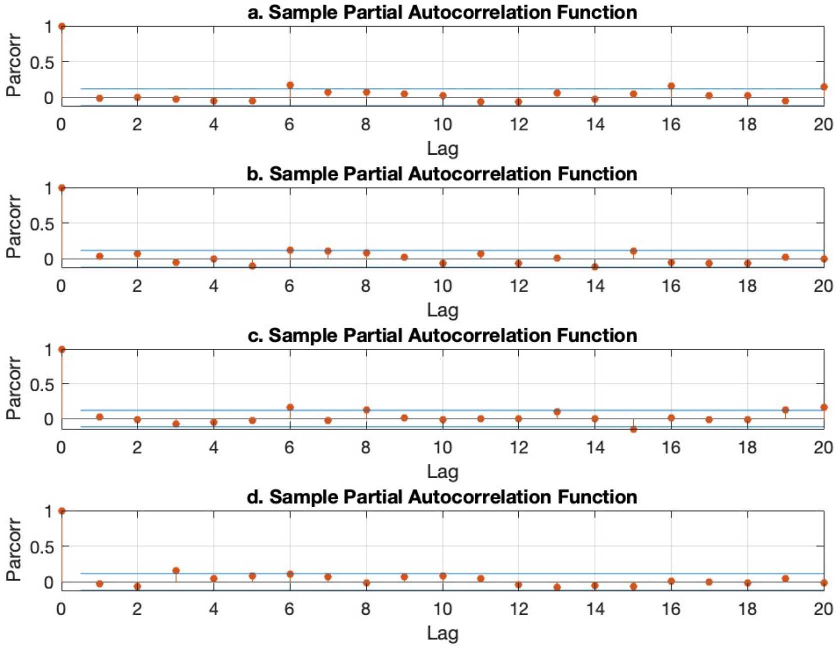

Figure A.16: this figure reports the partial autocorrelation functions for the residuals of the fourth (a.), the fifth (b.), the sixth (c.), and the seventh (d.) variable of the system. Notably, there is no partial autocorrelation in the residuals considered, as all the partial autocorrelations fall inside the 95% confidence interval.

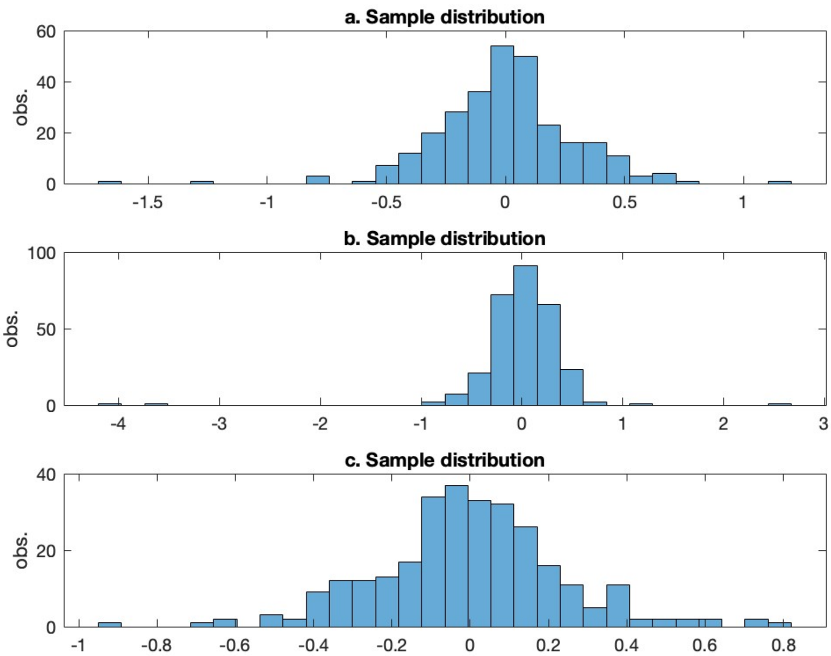

Figure A.17: this figure reports the sample distribution of the residuals of the first (a.), the second (b.), and the third (c.) variable of the system.

Notably, both Figure A.17 and Figure A.18 depict residuals that appear to be normally distributed or very close to it. Given the relatively small sample size, which may not fully capture the population distribution, these observations provide justification for assuming normality within the context of this analysis.

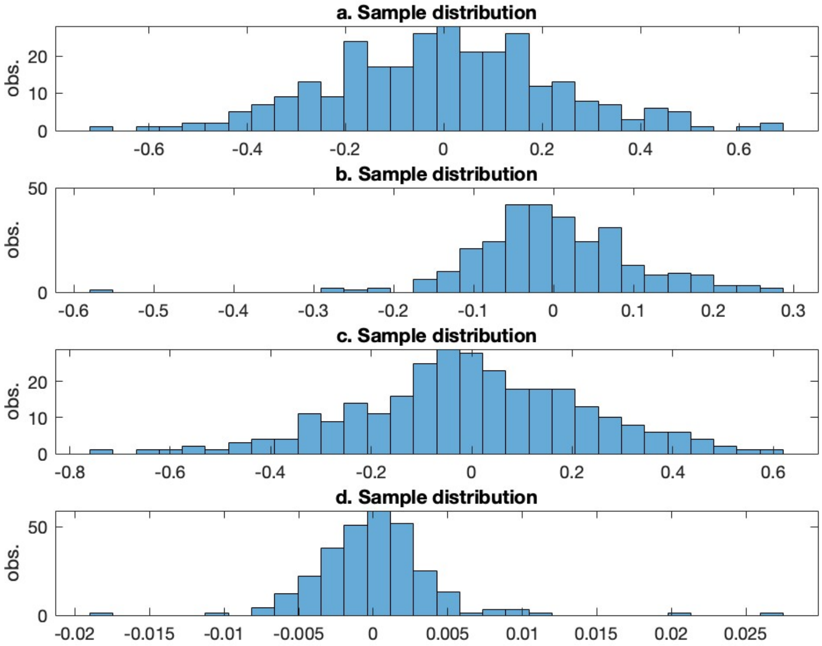

Figure A.18: this figure reports the sample distribution of the residuals of the fourth (a.), the fifth (b.), the sixth (c.), and the seventh (d.) variable of the system.

The informal residuals diagnostic conducted shows that the residuals are both independent with respect to themselves, and are normally distributed with zero mean. This validates the use of ML to estimate the parameters under normality of the error terms.