Department of Economics and Finance

From Gacha to Gambling: A Comparative Analysis of House Advantage in Digital and Traditional Games

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Introduction

Many legislative bodies have argued that Random Reward Systems (RRMs) in video games constitute gambling and should, therefore, be regulated as such. However, little progress has been made due to difficulties in defining which elements of RRMs qualify as gambling. This lack of clarity presents regulators with two potential risks. Firstly, future regulations might adopt definitions that are too broad, inadvertently stifling innovation in the gaming industry or restricting game mechanics that do not pose a genuine gambling risk. Conversely, regulations could be too narrow, targeting specific implementations that developers could easily circumvent, allowing similar harmful practices to remain unregulated.

This thesis synthesized existing literature to define RRMs and revealed a general consensus that RRMs should be regarded as a form of gambling. Generally, gambling is legally defined as any activity that involves staking something of value on an outcome determined predominantly by chance. Most games avoid this classification by offering rewards with no real-world value, arguing that players engage in them for self-gratification rather than monetary gain. However, research shows that even without a cash-out option, players can still incur significant financial losses. This thesis supports the view that RRMs in video games can be as harmful as traditional gambling. To evaluate their impact, we introduce the concept of House Advantage.

House Advantage is the mathematical edge that a casino or, in this case, the game operator has over the player in any given game. This thesis posits that House Advantage can serve as an objective framework for measuring the harm caused by RRMs, as it represents the percentage of each bet that the operator expects to retain as profit over time. This metric not only provides insight into the risks posed by such systems but also allows for meaningful comparisons between different gambling activities.

By comparing RRMs to other gambling games, this thesis aims to bring greater clarity to the ongoing discussion and provide legislators with a more objective metric(House Advantage) on which to base their decisions.

Chapter 1

Background

Recently, the gaming industry has experienced outstanding and continuous growth, becoming bigger than the movie and music industries combined[1].

This assures investors and developers alike of the financial prowess of this industry and incentivizes them to pour increasing amounts of money into their development. Unsurprisingly, video game development budgets have soared, with costs now reaching hundreds of millions of dollars for development and marketing[2]. This surge in funding has led to great innovations in the gaming landscape but has also placed considerable pressure on developers to deliver exceptional results, especially in the financial sense. It isn't easy to justify spending hundreds of millions of investor funding on an untested product without any guarantee of profit and, therefore, game developers have resorted to several controversial tactics to answer the amounting commercial pressures of this expanding, hyper-competitive

[1] "The Gaming Industry: A Behemoth With Unprecedented Global Reach" `https://www.forbes.com/councils/forbesagencycouncil/2023/11/17/the-gaming-industry-a-behemoth-with-unprecedented-global-reach/`

[2] "Why Have Video Game Budgets Skyrocketed In Recent Years¿' `https://www.forbes.com/sites/quora/2016/10/31/why-have-video-game-budgets-skyrocketed-in-recent-years/`

market, such as “Random Reward Mechanisms“(RRMs)[10].

1.1 Random Reward Mechanism

RRMs are virtual processes that offer players a stochastic distribution of rewards in exchange for a fixed input. This stochastic nature means that RRMs are designed so that players bet their money on the uncertain outcome of a random program hoping to receive something of greater value[2]. The parallels to traditional gambling weren’t left unnoticed. Numerous studies[15, 24, 2] have shown the correlation between micro-transactions, “Internet Gaming Disorder (IGD)“, and gambling disorders across the world, demonstrating a positive relationship between maladaptive gambling and the amount of money spent in-game and, more worryingly, that children are also being exposed to such mechanisms[3]. Studies have also shown that 56.7% of games deemed suitable for those aged 4+ by Apple contained RRMs, as did 68.8% rated 9+ and 76.3% rated 12+ [4, 23] and that most games were wrongly labeled[22], failing to disclose their in-game usage.

Therefore regulators around the world started to express their concerns about the harm of such mechanisms on players, for instance, Belgium[3], U.K.[4], France[5], China[6], Australia[7] are considering taking regulatory action to limit the proliferation of these tools. However, due to varying definitions of gambling in the

[3] https://www.koengeens.be/news/2018/04/25/loot-boxen-in-drie-videogames-in-strijd-met-kansspelwetgeving

[4] https://www.gamblingcommission.gov.uk/about-us/page/virtual-currencies-esports-and-social-gaming-discussion-paper

[5] https://anj.fr/sites/default/files/2019-12/rapport-activite-Arjel%202018.pdf

[6] https://www.gov.cn/zhengce/zhengceku/2021-09/01/content_5634661.htm

[7] https://www.aph.gov.au/Parliamentary_Business/Committees/Senate/Environment_and_Communications/Gamingmicro-transactions/Report/c04

various jurisdictions, it is hard to define what parts of RRMs should be considered gambling. Game studios often use RRMs as an inexpensive way to offer players different experiences with the same game by surprising them with random variables that change the course of each session. An outright ban on RRMs could be devastating for the industry.

Broadly speaking,Nielsen et al. state that RRMs can be separated into four categories[13] based on how intertwined they are with the real-world economy, from the least intertwined(isolated) to the most intertwined(embedded) they are:

Type	**Resources**	**Reward**
I-I	Isolated(non purchasable)	Isolated(non sellable)
I-E	Isolated(non purchasable)	Embedded(sellable)
E-I	Embedded(purchasable)	Isolated(non sellable)
E-E	Embedded(purchasable)	Embedded(sellable)

They consider only E-E to be full gambling; E-I to be pseudo-gambling due to the inability to cash out, but requiring a participation fee; I-E to not be gambling despite the possibility of a paycheck due to the lack of a monetary stake; I-I to not be gambling due to being economically isolated items. Current regulations agree with this stance, since many countries, like Italy, define gambling as " ...any activity partaken for profit and whose win or loss is determined entirely or almost entirely by chance"[8]. Other governments in Europe, like the U.K.[9], Belgium[10] or Germany[11] follow similar regulations. To summarize, gambling is defined as (i)any game, (ii) for profit that (iii)requires a stake, and (iv)whose outcome is

[8]art. 721, Codice Penale

[9]Gambling Act 2005

[10]Gambling Act of 7 May 1999

[11]Section 3 (1) Sentences 1 and 2 of the Interstate Treaty on Gambling 2021 Glucksspielstaatsvertrag

entirely or mostly aleatory. These points also align with Griffith‘s characteristics that differentiate gambling from other risk-related behaviors[6].

Type	**Resources**	**Reward**	Criteria
I-I	Isolated(non purchasable)	Isolated(non sellable)	i,iv
I-E	Isolated(non purchasable)	Embedded(sellable)	i,ii,iv
E-I	Embedded(purchasable)	Isolated(non sellable)	i,iii,iv
E-E	Embedded(purchasable)	Embedded(sellable)	i,ii,iii,iv

Under this definition, only E-E legally constitutes gambling. However, this notion was deemed too narrow by Drummond et al.[2], who proved that in-game items intrinsically have a monetary value that players can extract on grey markets where legal avenues are absent. Even if users were unable to cash out eventually, they would still suffer from gambling-related harms like financial ruin. Moreover, developers abuse decision-making cognitive biases and fallacies[20, 1] to make players spend more money than they would otherwise have spent by normalizing gambling[21], having gateway effect[17] and reinforcing them through behavioral conditioning[18]. Therefore, E-I RRMs should also be considered gambling since players lose real-life currency participating.

In this thesis, we will quantify this financial harm as House Advantage and compare the House Advantage of a subset of E-I,“ Gacha Games“, to more traditional gambling activities to ascertain whether they are equally or more unfavorable than the latter and, therefore, equally harmful.

1.2 House Advantage

The house advantage is a numerical index of the unfavorability of a wager.

A wager (or bet) is described by a pair (B,X), where:

- B, X are jointly distributed random variables;
- B denotes the amount bet (i.e., the amount placed at risk);
- X denotes the gambler‘s profit (positive, negative, or zero);

Gamblers cannot lose more than he bets, hence:

- $\mathbb{E}[B] < \infty$;
- $\mathbb{E}[X1_{\{X \neq k\}}] = \mathbb{E}[B|X \neq 0] \cdot \mathbb{P}(X \neq 0) > 0$;
- $\mathbb{E}[|X|] < \infty$.

The House Edge can be defined in two ways:

$$H_0(X, B) := \frac{-\mathbb{E}[X]}{\mathbb{E}[B]} = \frac{-\mathbb{E}[X]}{\mathbb{E}[B|X \neq 0] \cdot \mathbb{P}(X \neq 0) + \mathbb{E}[B|X = 0] \cdot \mathbb{P}(X \neq 0)} \quad (1.1)$$

Or

$$H(X,B) := \frac{-\mathbb{E}[X]}{\mathbb{E}[B1_{X \neq 0}]} = \frac{-\mathbb{E}[X]}{\mathbb{E}[B \mid X \neq 0] \cdot \mathbb{P}(X \neq 0)} \quad (1.2)$$

Note that

- $\mathbb{E}[X]$ is the gambler‘s expected profit. But in casino games ,it is usually negative, so $-\mathbb{E}[X]$ represents the gambler‘s expected loss;
- $\mathbb{E}[B]$ is the gambler‘s expected amount bet;
- $\mathbb{E}[B1_{X \neq 0}] = \mathbb{E}[B \mid X \neq 0] \cdot \mathbb{P}(X \neq 0) > 0$ is the gambler‘s expected amount of action, that is the gambler‘s expected amount bet except in the case of a push (i.e., tie). A push provides no action.

We can find the house edge of a game from a single bet since let $(B_1, X_1), \ldots, (B_n, X_n)$ be independent and identically distributed(i.i.d.) random vectors whose common distribution is that of (B, X) representing the results of the independent repetitions of the original wager. Then

$$\frac{-(X_1 + \ldots + X_n)}{B_1 + \ldots + B_n}$$

represents the ratio of the player's cumulative loss after n such wagers to his total amount bet, while

$$\frac{-(X_1 + \ldots + X_n)}{B_1 \mathbf{1}_{\{X_1 \neq 0\}} + \ldots + B_n \mathbf{1}_{\{X_n \neq 0\}}}$$

represents the ratio of the player's cumulative loss after n such wagers to his total amount of action.

By the Law of Large Numbers, we have:

$$\frac{-(X_1 + \ldots + X_n)}{B_1 + \ldots + B_n} = \frac{\frac{-(X_1+\ldots+X_n)}{n}}{\frac{B_1+\ldots+B_n}{n}} \underset{n\to\infty}{\longrightarrow} \frac{-\mathbb{E}[X]}{\mathbb{E}[B]} = H_0(X, B) \quad \text{almost surely} \tag{1.3}$$

and

$$\frac{-(X_1 + \ldots + X_n)}{B_1 \mathbf{1}_{\{X_1 \neq 0\}} + \ldots + B_n \mathbf{1}_{\{X_n \neq 0\}}} = \frac{\frac{-(X_1+\ldots+X_n)}{n}}{\frac{B_1 \mathbf{1}_{\{X_1 \neq 0\}}+\ldots+B_n \mathbf{1}_{\{X_n \neq 0\}}}{n}} \underset{n\to\infty}{\longrightarrow} \frac{-\mathbb{E}[X]}{\mathbb{E}[B_1 \mathbf{1}_{\{X_1 \neq 0\}}]} = H(X, B) \quad \text{almost surely.} \tag{1.4}$$

More specifically:

- $H_0(X, B)$ is the long-term ratio of the gambler's cumulative loss to his total amount bet;
- $H(X, B)$ is the long-term ratio of the gambler's cumulative loss to his total amount of action.

Let us now consider the first round in which the outcome is not a tie. That is, let us define

$$N = \min\{n \geq 1 | X_n \neq 0\}$$

This is a stopping time, since to know the first win or loss, we need information up to round k, such that:

$$\{\tau = k\} = \{X_1 = 0, X_2 = 0, \ldots, X_{k-1} = 0, X_k \neq 0\} \tag{1.5}$$

Moreover, since the single wagers are independent, this random time is a geometric random variable of parameter $\mathbb{P}(X \neq 0)$. Indeed given $k \in \mathbb{N}$ we have

$$\begin{aligned} \mathbb{P}(N = k) = \mathbb{P}(X_1 = X_2 = \ldots = X_{k-1} = 0, X_k \neq 0) &\overset{indep.}{=} \\ \overset{indep.}{=} \mathbb{P}(X_k \neq 0) \cdot \prod_{i=1}^{k-1} &\overset{id.distr.}{=} \\ \overset{id.distr.}{=} \mathbb{P}(X \neq 0) \cdot \mathbb{P}(X = 0)^{k-1} = \mathbb{P}(X \neq 0) \cdot (1 - \mathbb{P}(X \neq 0))^{k-1} \end{aligned} \tag{1.6}$$

So $N \sim Geom(\mathbb{P}(X \neq 0))$ and by computing the house advantage of the sequence of wagers considering up to time N, that is up to the first round in which the outcome is not a tie, we can see that:

$$\begin{aligned} H_0(B_1 + \ldots + B_n, H_1 + \ldots + H_n) = H_0(B, X) \\ H_0(B_n, X_n) = H(B, X) \end{aligned} \tag{1.7}$$

Since $N \sim Geom(\mathbb{P}(X \neq 0))$, we have $\mathbb{P}(X < \infty) = 1$ and hence it has meaning to consider the random vector $\mathbb{P}(B_n, X_n)$.

$$\begin{aligned} H_0(B_1 + \ldots + B_n, H_1 + \ldots + H_n) = H_0(\sum_{i=1}^{N} B_1, \sum_{i=1}^{N} X_1) = \\ = \frac{-\mathbb{E}[\sum_{i=1}^{N} X_1]}{\mathbb{E}[\sum_{i=1}^{N} B_1]} = \frac{-\mathbb{E}[N]\mathbb{E}[X]}{-\mathbb{E}[N]\mathbb{E}[B]} = \frac{-\mathbb{E}[X]}{-\mathbb{E}[B]} = H_0(B, X) \end{aligned} \tag{1.8}$$

Note that $\{N = i\} = \{X_1 = \ldots = X_{i-1} = 0, X_i \neq 0\}$ and

$$\mathbb{E}[1_{\{X_1 = \ldots = X_{i-1} = 0, X_i \neq 0\}}] = 1 \cdot \mathbb{P}(X_1 = \ldots = X_{i-1} = 0) \overset{i.i.d.}{=} \mathbb{P}(X = 0)^{i-1}$$

Consider now a function $f(b,x)$. We have

$$\begin{aligned}
\mathbb{E}[f(B_N,X_N)] &= \sum_{i=1}^{\infty}\mathbb{E}[f(B_i,X_i)1_{\{N=i\}}] = \\
&= \sum_{i=1}^{\infty}\mathbb{E}[f(B_i,X_i)1_{\{X_1=\ldots=X_{i-1}=0,X_i\neq 0\}}] \overset{ind}{=} \\
&\overset{ind}{=} \sum_{i=1}^{\infty}\mathbb{E}[f(B_i,X_i)1_{\{X\neq 0\}}]\mathbb{E}[1_{\{X_1=\ldots=X_{i-1}=0,X_i\neq 0\}}] = \\
&= \sum_{i=1}^{\infty}\mathbb{E}[f(B_i,X_i)1_{\{X\neq 0\}}]\mathbb{P}(X=0)^{i-1} \overset{id.distr}{=} \\
&\overset{id.distr}{=} \sum_{i=1}^{\infty}\mathbb{E}[f(B,X)1_{\{X\neq 0\}}]\mathbb{P}(X=0)^{i-1}
\end{aligned} \tag{1.9}$$

Since for $a \in (0,1)$

$$\sum_{j=0}^{\infty} a^j = \frac{1}{1-a},$$

We have

$$\sum_{i=1}^{\infty}\mathbb{P}(X=0)^{i-1} \overset{j=i-1}{=} \sum_{i=1}^{\infty}\mathbb{P}(X=0)^{j} = \frac{1}{1-\mathbb{P}(X=0)} = \frac{1}{\mathbb{P}(X\neq 0)} \tag{1.10}$$

Hence

$$\begin{aligned}
\mathbb{E}[f(B_N,X_N)] &= \sum_{i=1}^{\infty}\mathbb{E}[f(B,X)1_{\{X\neq 0\}}]\mathbb{P}(X=0)^{i-1} = \\
&= \mathbb{E}[f(B,X)1_{\{X\neq 0\}}]\sum_{i=1}^{\infty}\mathbb{P}(X=0)^{i-1} = \\
&= \mathbb{E}[f(B,X)1_{\{X\neq 0\}}]\cdot\frac{1}{\mathbb{P}(X\neq 0)} = \\
&= \mathbb{E}[f(B,X)|X\neq 0]
\end{aligned} \tag{1.11}$$

If we take as f the function $f(b,x)=x$ we have

$$\begin{aligned}
\mathbb{E}[X_N] &= \mathbb{E}[f(B_N,X_N)] = \mathbb{E}[f(B,X)|X\neq 0] = \mathbb{E}[X|X\neq 0] = \\
&= \frac{\mathbb{E}[X\cdot 1_{\{X\neq 0\}}]}{\mathbb{P}(X\neq 0)} = \frac{\mathbb{E}[X\cdot 1_{\{X\neq 0\}}]+\mathbb{E}[0\cdot 1_{\{X=0\}}]}{\mathbb{P}(X\neq 0)} = \\
&= \frac{\mathbb{E}[X\cdot 1_{\{X\neq 0\}}]+\mathbb{E}[X\cdot 1_{\{X=0\}}]}{\mathbb{P}(X\neq 0)} = \\
&= \frac{\mathbb{E}[X\cdot(1_{\{X\neq 0\}}+1_{\{X=0\}})]}{\mathbb{P}(X\neq 0)} = \frac{\mathbb{E}[X]}{\mathbb{P}(X\neq 0)}
\end{aligned} \tag{1.12}$$

If we take as f the function $f(b,x) = b$ we have

$$\begin{aligned} \mathbb{E}[B_N] = \mathbb{E}[f(B_N, X_N)] = \mathbb{E}[f(B,X)|X \neq 0] = \\ = \mathbb{E}[B|X \neq 0] = \frac{\mathbb{E}[B \cdot 1_{\{X\neq 0\}}]}{\mathbb{P}(X \neq 0)} = \frac{\mathbb{E}[B]}{\mathbb{P}(X \neq 0)} \end{aligned} \tag{1.13}$$

So

$$\begin{aligned} H_0(B_N, X_N) = \frac{-\mathbb{E}[X_N]}{-\mathbb{E}[B_N]} = \frac{\frac{-\mathbb{E}[X]}{\mathbb{P}(X\neq 0)}}{\frac{\mathbb{E}[B]}{\mathbb{P}(X\neq 0)}} = \\ = \frac{-\mathbb{E}[X]}{\mathbb{E}[B \cdot 1_{\{X\neq 0\}}]} = H(B,X) \end{aligned} \tag{1.14}$$

From this proposition, we deduce the following considerations:

- if a push is regarded as a conclusive outcome of the bet, then $B_1 + ... + B_N$ units have been bet by time N and H_0 is the appropriate definition of house advantage. We describe H_0 as house advantage with pushes included.

- if a push is regarded as merely a delay in the eventual resolution of the bet, Only B_N units have been bet by time N, and H is the appropriate definition of house advantage. We describe it as house advantage with pushes excluded.

- A game is more favorable if it has a smaller house advantage.

To reduce unwanted variables, this thesis will not consider subjective games like sports betting or poker and will analyze only games with theoretical probabilities, like roulette and one gacha game, Genshin Impact.

More specifically, in this thesis, we will see:

- Roulette(European and American Variant);

- Slot-Machine(Liberty Bell and Mills);

- Keno(8-spot and 10-spot rate).

- Genshin Impact(Limited Character Banner and Limited Weapon Banner

1.3 Gacha Games

Despite the spread of RRM, few game genres can be said to be defined by their monetization system as Gacha games. In this context, this genre is the most appropriate case study for this research.

Gacha games are usually free-to-play games(F2P) and owe their name to their roulette-like monetization systems. Users pay with in-game currency to draw a character or item they want from a predetermined set of virtual items of varying degrees of rarity. Players who fail to obtain their coveted item can buy more attempts with real-world money. However, the items obtained are non-transferable and cannot be resold (E-I). The main difference between Gacha and other genres is that it motivates players to pay money for an emotional payoff that comes from relationality rather than financial by creating a bond between the user and the desired character or item[14].

But evaluating house advantage requires information that developers rarely release truthfully[12], except in China, where disclosure is required by law. As such, this thesis will focus mainly on Chinese games, and among the various examples of Chinese Gacha games, Genshin Impact will serve as the primary case study, given its distinction as one of the most profitable and successful Gacha games to date; becoming the biggest gaming sensation of 2020, reaching a revenue of 1 billion dollars in just 6 months[13] and 3.6 billion dollars in its first year[14], setting new standards for other developers. Many subsequent games[15] released after Genshin's global success present some uncanny resemblance to Genshin Impact. Still, for

[12] `https://www.businesskorea.co.kr/news/articleView.html?idxno=209008`

[13] `https://sensortower.com/blog/genshin-impact-one-billion-revenue`

[14] `https://www.pocketgamer.biz/genshin-impact-surpasses-36-billion-revenue-ahead-of-second-anniversary/`

[15] Wuthering Waves, Tower of Fantasy, Girl‘s Frontier 2, Reverse:1999, etc...

this thesis, only two features are relevant:

- presence of a "pity system", which offers players a guarantee of obtaining their desired item after a set number of pulls[16].
- "Constellations", which are sets of sequential upgrades to Characters or Items by obtaining duplicates[17].

Therefore, this thesis will focus on Genshin Impact and its monetization methods. As the largest Gacha game on the market, Genshin Impact is a model that many of its competitors emulate, making it an ideal subject for an in-depth analysis of the genre.

1.3.1 Genshin Impact

Genshin Impact is an open-world adventure RPG[18]. As a F2P game, players can acquire and play it free of charge. This has enabled a wider range of player segments to try the game[9], reaching more than 202 million lifetime downloads[19]. This huge population and widespread geographical distribution assures generalisability and cross-cultural validity of our findings.

[16]https://sg.news.yahoo.com/genshin-impact-how-does-pity-system-work-051743468.html?guccounter=1&guce_referrer=aHR0cHM6Ly93d3cuZ29vZ2xlLmNvbS8&guce_referrer_sig=AQAAAGs3fopMgKCZFKwpOoQ8gh8eGWw57dsL8cA2zhkmxGzBMCB8iwW4YGWc4qqrCo0zE1rRrLo0M8cJmhvl6UtO4mu9QQTL2P8JM_m39k5wFcn2UcYCo327vokrilnzaUzBhvYKjdxFjvH2P4sM2pNe_uY08eSUy6Buvl U39l6KAwow

[17]https://www.ign.com/wikis/genshin-impact/Constellations_-_What_Are_They_and_How_to_Get_Them

[18]https://genshin.hoyoverse.com/it/

[19]https://www.statista.com/statistics/1251724/genshin-impact-number-of-downloads-worldwide/#:~:text=Released%20in%20September%202020%2C%20Genshin,of%2032%20million%20U.S.%20dollars.

The game encourages in-game purchases by locking some items behind Gacha mechanisms. Genshin Impact has three kinds of "banners"(sets) that players can use one game token to pull once from: Standard Banner, Limited Character Banner, and Limited Weapon Banner. The rarest items are the 5-star items, which have a 0.6% drop rate across the banners, but this statistic may be misleading due to the presence of soft/hard pities. A soft pity means that after a certain number of tries, the chances of winning go up till reaching 100%, ensuring players a 5-star item; a hard pity instead guarantees a specific 5-star item if, after a certain number of wins, a player fails to get his desired item[20]. Mihoyo, as of date, has never explained how the soft pity works, but players and other industry players determined that after m draws, the probability of winning increases by 5.85% until reaching 100%.

Hence every pull is dependent on the previous ones due to the pity system, which increases your chances of pulling a 5-star over time. Here is the probability distribution of the soft-pity:

$$X = \begin{cases} n & \text{with } p = (1-p)^{n-1} \cdot p \quad \text{for } 0 < n \leq m, \\ n & \text{with } p = (1-p)^{m}[p + (n-m)i] \cdot \prod_{k=1}^{n-m+1}(1 - p - i \cdot k) \quad \text{for } m < n \end{cases} \tag{1.15}$$

However, once you win a 5-star character, the soft pity system resets, making all future pulls independent of the previous ones. Each new pull after that starts fresh, with no carry-over from past pulls. So we can consider $\mathbb{E}[X]$ as the expected n^{th} round of winning the soft pity.

The hard pity system introduces an additional dependency on failed wins. After each failed attempt to obtain the desired limited 5-star character, the probability

[20] `https://ys.mihoyo.com/main/news/public`

of getting that character increases sharply until it is guaranteed on the next 5-star pull. The probability distribution for hard pity follows:

$$X_2 = \begin{cases} \mathbb{E}[X] & \text{with } p = p_1, \\ \vdots \\ n\mathbb{E}[X] & \text{with } p = p_n \end{cases} \tag{1.16}$$

Once the limited 5-star character is obtained, the hard pity system also resets, making future attempts independent of previous ones. Each new 5-star win starts from the baseline probability, with no carry-over from prior attempts. Therefore, the expected value $\mathbb{E}[X_2]$ represents the number of wins needed to find the limited 5-star character through the hard pity system.

Chapter 2

House Advantage

To measure the house edge, we need to find the expected gain and expected cost. Below, we measure the expected house edge of the games considered.

2.1 Roulette

A roulette has 36 numbers and n zeroes depending on the type; a player can bet on any subset of size m of the set of 36 + n numbers for $m = 1, 2, 3, 4, 6, 12, 18$ and pays $\frac{36}{m} - 1$ to 1 if a number in that subset appears, there is no possibility of a push, hence:

$$X = \begin{cases} b \cdot \left(\frac{36}{m} - 1\right) & \text{with p} \quad \frac{m}{36+n} \\ -b & \text{with p} \quad 1 - \frac{m}{36+n} \end{cases}$$

$$\mathbb{E}[X] = b \cdot \left(\frac{36}{m} - 1\right) \cdot \frac{m}{36+n} - b \cdot \left(1 - \frac{m}{36+n}\right) =$$

$$\mathbb{E}[X] = b \cdot \left[\frac{36}{36+n} - \frac{m}{36+n} - 1 + \frac{m}{36+n}\right]$$

$$\mathbb{E}[X] = b \cdot \frac{-n}{36+n}$$

As such, if we considered European Style(n=1) and American Style(n=2) and b=1 for ease, their house edge would be respectively $\sim -2.7\%$ and $\sim -5.2\%$.

2.2 Slot Machine

Slot machines were first developed in San Francisco in 1898 and have evolved substantially over time. In this paper, we will only consider some of the classics, like the first recognized 3-wheeled, 10-stop "Liberty Bell" slot machine and the 3-wheeled, 20-stop Mills mechanical slot machine.

Players insert a token into the machine, causing the reels to spin independently. When they come to rest, one symbol on each reel is visible, and the three resulting symbols determine the number of coins paid out to the player according to the pay table. For our example, those are:

reel 1	reel 2	reel 3	payout
bell	bell	bell	20
♡	♡	♡	16
♢	♢	♢	12
♠	♠	♠	8
Ω	Ω	★	4
Ω	Ω	not ★	2

(a) "Liberty Bell" Payout

reel 1	reel 2	reel 3	payout
7	7	7	200
bar	bar	bar	150
melon	melon	melon	150
melon	melon	bar	150
bell	bell	bell	18
bell	bell	bar	18
plum	plum	plum	14
plum	plum	bar	14
orange	orange	orange	10
orange	orange	bar	10
cherry	cherry	anything	5
cherry	not cherry	anything	2

(b) "Mills" Payout

Source: Ethier[5] pages 29-431

Fig. 2.1: Payouts

Evaluating a machine's house advantage also requires information about the probability of each combination. Each machine has a different distribution of symbols across its wheels, but there are at most s^w possible combinations, and only few of those actually offer a net gain to players. In our case studies, those are:

symbol	reel 1	reel 2	reel 3
bell	1	1	2
♡	1	1	1
♢	1	1	3
♠	2	2	2
Ω	5	5	0
★	0	0	2
total	10	10	10

(a) "Liberty Bell"

symbol	reel 1	reel 2	reel 3
7	1	1	1
melon	1	1	1
bar	1	1	1
bell	1	5	8
plum	5	3	3
orange	5	4	2
cherry	3	6	0
lemon	4	0	5
total	21	21	21

(b) "Mills"

Source: Ethier[5] pages 29-432

Fig. 2.2: Distributions

With these two points, it is now possible to determine the expected payout for each combination of each machine:

reel 1	reel 2	reel 3	payout	number of ways	product
bell	bell	bell	20	$1 \cdot 1 \cdot 2 =$ 2	40
♡	♡	♡	16	$1 \cdot 1 \cdot 1 =$ 1	16
♢	♢	♢	12	$1 \cdot 1 \cdot 3 =$ 3	36
♠	♠	♠	8	$2 \cdot 2 \cdot 2 =$ 8	64
Ω	Ω	★	4	$5 \cdot 5 \cdot 2 =$ 50	200
Ω	Ω	not ★	2	$5 \cdot 5 \cdot 8 =$ 200	400
total				264	756

(a) "Liberty Bell" Expected Payout

reel 1	reel 2	reel 3	payout	number of ways	product
7	7	7	200	$1 \cdot 1 \cdot 1 =$ 1	200
bar	bar	bar	150	$1 \cdot 1 \cdot 1 =$ 1	150
melon	melon	melon	150	$1 \cdot 1 \cdot 1 =$ 1	150
melon	melon	bar	150	$1 \cdot 1 \cdot 1 =$ 1	150
bell	bell	bell	18	$1 \cdot 5 \cdot 8 =$ 40	720
bell	bell	bar	18	$1 \cdot 5 \cdot 1 =$ 5	90
plum	plum	plum	14	$5 \cdot 3 \cdot 3 =$ 45	630
plum	plum	bar	14	$5 \cdot 3 \cdot 1 =$ 15	210
orange	orange	orange	10	$5 \cdot 4 \cdot 2 =$ 40	400
orange	orange	bar	10	$5 \cdot 4 \cdot 1 =$ 20	200
cherry	cherry	anything	5	$3 \cdot 6 \cdot 20 =$ 360	1,800
cherry	not cherry	anything	2	$3 \cdot 14 \cdot 20 =$ 840	1,680
total				1,369	6,380

(b) "Mills" Expected Payout

Source: Ethier[5] pages 29-431

Fig. 2.3: Expected Payouts

By summing the expected payouts for each combination, we find the expected payout for each machine:

$$\begin{aligned}
\mathbb{E}[R_l] &= 20\left(\frac{2}{1000}\right) + 16\left(\frac{1}{1000}\right) + 12\left(\frac{3}{1000}\right) + \\
&\quad + 8\left(\frac{8}{1000}\right) + 4\left(\frac{50}{1000}\right) + 2\left(\frac{200}{1000}\right) = 0.756 \\
\mathbb{E}[R_m] &= 200\left(\frac{1}{8000}\right) + 150\left(\frac{3}{8000}\right) + 18\left(\frac{45}{8000}\right) + \\
&\quad + 14\left(\frac{60}{8000}\right) + 10\left(\frac{60}{8000}\right) + 5\left(\frac{360}{8000}\right) + 2\left(\frac{840}{8000}\right) = 0.7975 \\
H_0(R_l) &= 1 - 0.756 = 0.244 \\
H_0(R_m) &= 1 - 0.7975 = 0.2025
\end{aligned}$$

2.3 Keno

Keno is a lottery-style game where players pay a fee to choose M numbers without replacement from a set $\{1, 2, \ldots, S-1, S\}$. The casino then chooses N numbers without replacement from the same set, and based on how many K matches a player gets, he can expect different payouts.

The maximum possible combinations are $\binom{S}{N}$, and the probability of having K matches is $\binom{M}{K}\binom{S-M}{N-K}$. Hence it is a hypergeometric distribution:

$$\mathbb{P}_m(k) = \frac{\binom{M}{K}\binom{S-M}{N-K}}{\binom{S}{N}}$$

The most popular versions of this game are the 8-spot and 10-spot tickets, where players choose 8 or 10 numbers from $\{1,\ldots,80\}$, and the house chooses 20 numbers in both versions. The expected probability of matching k for each style is summarized as such:

K	$\mathbb{P}_m(k)$
0	0.0883
1	0.226
2	0.328
3	0.215
4	0.081
5	0.018
6	0.002
7	0.00016
8	0.0000043

(a) 8-spot

K	$\mathbb{P}_m(k)$
0	0.046
1	0.1796
2	0.295
3	0.0.267
4	0.147
5	0.0.051
6	0.0115
7	0.0016
8	0.000135
9	0.00000612
10	0.0000001122

(b) 10-spot

Table 2.1: Probability of k matches

Using the average payout of some Las Vegas Casinos(Appendix A.3), we found the average house edge of Keno are -29.673% and -30.776%, respectively.

2.4 Genshin Impact

Each of its three banners presents slight variations in pities that change each payout and cost, which we will analyze individually. All results were then compared to empirical data gathered by players[1] and from our own simulations[2].

2.4.1 Cost of Entry

A token costs 160 “Primogems“, the premium currency in the game. Players can obtain this currency by playing the game or spending real-life money. A player can buy this currency in the in-game shop at these values:

Cost	**Primogems**	**Cost of 1 pull**
1€	60	2.67€
6€	330	2.91€
18€	1090	2.64€
35€	2240	2.50€
60€	3880	2.47€
100€	8080	1.98€

Here, we can see the economy of scale at work. The game incentivizes players to buy more Primogems by offering them at lower prices. As such, we will assume the cost of a pull to be around 2€.

Moreover, from a player-made aggregator[3], which tracked the pulling history of hundreds of thousands of players, we can see that the vast majority of players only pull for 1 copy of either weapon or character, more precisely 90% and 70% of players get 1 copy of either weapon or character. As such, we will assume that

[1] `https://paimon.moe/wish/tally`

[2] `https://github.com/uhuybubb/simulations`

[3] `https://paimon.moe/wish/tally?id=400069`

most players only draw for 1 character and 1 weapon from both limited banners, commonly referred to by players as 0+1.

2.4.2 Standard Character Banner

Every pull in the standard character banner has a 0.6% base chance of yielding a 5-star item between 10 weapons and 7 characters. After the 73rd pull, a soft pity kicks in. Each pull after has a 5.85% higher chance of success until reaching 100% at the 90th pull.

As such, if we substitute 73 as m in equation 2.4.4:

$$X = \begin{cases} n & \text{with } p = (1-p)^{n-1} \cdot p \quad \text{for } 0 < n \leq 73, \\ n & \text{with } p = (1-p)^{73}[p + (n-73)i] \cdot \prod_{k=1}^{n-74}(1 - p - i \cdot k) \quad \text{for } 73 < n \end{cases}$$

From table 2.3 we can see that $\mathbb{E}[X] \approx 63$, since partial pulls are impossible, all results have been rounded by excess.

However, after obtaining a 5-star item, there is a 50/50 chance of it being either a weapon or a character. Furthermore, there is another 1/7 and 1/10 chance of getting a specific character or weapon.

With these probabilities, if we consider that there is a 50/50 split of getting either a weapon or a character and that there is a further 1/7 and 1/10 chance of getting any specific character or weapon, we found that

$$\mathbb{E}[C] = \mathbb{E}[X] \cdot 7 \cdot 2 \approx 882$$
$$\mathbb{E}[W] = \mathbb{E}[X] \cdot 10 \cdot 2 \approx 1260$$

This means that, on average, for a specific character or weapon, a person would need to make 882 and 1260 pulls, respectively. And for 2€ per pull, it would cost around 1764€ and 2520€. Moreover, a character's and weapon‘s true potentials are locked unless a player draws 7 copies and 5 copies, respectively; this pumps up the numbers to 12348€ and 12600€

Players considered these prices excessive and deemed this banner the worst of the three, something to be actively avoided. Few players spend money on this banner; as such, this banner will not be considered for our thesis.

2.4.3 Limited Character Banner

Every pull in the standard character banner has a 0.6% base chance of yielding any 5-star item between 7 standard characters or 1 limited character. In the Limited Character banner, the soft pity guarantees a 5-star item before the 90th pull. For every win, the user has a 50/50 chance of getting the limited or any standard character. There is also a hard pity that ensures that the player gets the limited character at most at his second win; the first successful pull will be either the limited character or a standard one, but if he gets a standard character, then his next 5-star character will be the limited character.

1st	2nd	prob
Success	\\	0.5
Standard	Success	0.5

So, if we define X as the number of rounds needed to get any 5-star character and X_2 as the number of wins needed to get the limited character, then $\mathbb{E}[X] \approx 63$, as it follows a similar distribution to the standard banner 2.4.4 and:

$$X_2 = \begin{cases} 1\mathbb{E}[X] \text{ with } p = \frac{1}{2}, \\ 2\mathbb{E}[X] \text{ with } p = \frac{1}{2} \end{cases} \tag{2.1}$$

Hence $\mathbb{E}[X_2] = \frac{1}{2}\mathbb{E}[X] + \mathbb{E}[X] = \frac{3}{2}\mathbb{E}[X] \approx 95$ pulls, or around 190€. This number is much more modest than the previous one, even if we consider getting a limited character to its full potential would cost only 1316€.

2.4.4 Limited Weapon Banner

Every pull in the limited weapon banner has a 0.6% base chance of yielding any 5-star item between 10 standard weapons and 2 limited weapons. The Limited Weapon banner has a soft pity that guarantees a 5-star item before the 80th pull. After the 63rd pull, a soft pity kicks in. Each pull then has a 5.85% higher chance of success, until reaching 100%. By substituting 63 as m in equation 2.4.4

$$W = \begin{cases} n \quad \text{with } p = (1-p)^{n-1} \cdot p \quad \text{for } 0 < n \leq 63, \\ n \quad \text{with } p = (1-p)^{63}[p + (n-63)i] \cdot \prod_{k=1}^{n-64}(1 - p - i \cdot k) \quad \text{for } 63 < n \leq 80 \end{cases}$$

Hence(2.4):

$$\mathbb{E}[W] \approx 55$$

There is also a hard pity that ensures that the player gets the limited weapon he wants at most at his third win; the first 5-star weapon a player gets has a 25% chance of being a standard weapon and 37.5% chance of being either limited weapon; if the player fails to get his desired weapon the first time, he then has a 50% chance of winning either limited weapon; if he fails again he then has a 100% chance of getting it the third time.

1st	2nd	3rd	prob
Success	\\	\\	0.375
Other	Success	\\	0.14063
Standard	Success	\\	0.125
Other	Standard	Success	0.09375
Standard	Other	Success	0.125
Other	Other	Success	0.14063

If we consider W as the number of rounds needed to win, W_1, W_2 and W_3 as the probability of finding the desired weapon the first, second and third win respectively, then:

$$W_S = \begin{cases} 1\mathbb{E}[W_1] & \text{with } p = 0.375 \\ 2\mathbb{E}[W_2] & \text{with } p = 0.26563 \\ 3\mathbb{E}[W_3] & \text{with } p = 0.35938 \end{cases} \tag{2.2}$$

Hence the $\mathbb{E}[W_S] = (1 \cdot 0.375 + 2 \cdot 0.26563 + 3 \cdot 0.35938) \cdot \mathbb{E}[W] \approx 109$ pulls, or around 218€. For 5 copies, a player would need to spend 1090€.

2.4.5 Expected Gain

Mihoyo, the developers of Genshin Impact, state in their terms of service that users “...shall neither transfer or otherwise make your Account information available to third parties, nor use other User(s)’ Account(s) at any time“. But, just as Drummond predicted, “...where legitimate marketplaces are absent, spontaneous grey markets have emerged (where possible) to allow virtual item trading. “

In the online space, multiple third-party websites allow for the selling and purchasing of Genshin Impact accounts, such as “PlayerAuction“, “EBay“, “IG-Vault“ and many more. Unfortunately, items in Genshin Impact cannot be sold separately, sellers must sell their whole accounts with all their 5-star items as a bundle. Therefore, to find a baseline value for each individual item, we went online and recorded 250 transactions for a cumulative worth of 209254.46€[4]. We recorded the seller‘s asking price and assumed that to be the final price; we then counted all 5-star items in one‘s possession and divided the price by the total amount of items, finding the mean across accounts.

[4] `https://1drv.ms/x/s!Apq2XIWjrGXMhXFzpSHWFi57Xyjl?e=N4heJr`

On average, we determined that for any additional 5-star item, an account's value increases by a value of around ~ 20€.

Since $\mathbb{E}[X] = 63$ and cost per pull is 2€, the expected gain per win in the limited character banner is $\mathbb{E}[X_c] = 20 - 63 \cdot 2 = -106$€.

Hence by substituting $\mathbb{E}[X]$ with $\mathbb{E}[X_c]$ in equation 2.1, we find the expected gain when pulling for a specific limited 5-star character in the Limited Character banner:

$$X_l = \begin{cases} 1[X_c] \text{ with } p = \frac{1}{2}, \\ 2[X_c] \text{ with } p = \frac{1}{2} \end{cases} \quad \mathbb{E}[X_l] = -159\text{€} \tag{2.3}$$

The same can be applied for the Limited weapon banner since $\mathbb{E}[W] = 55$ and the cost per pull is 2€, the expected cost per win is 110€. Hence $\mathbb{E}[W_c] = -90$.

By substituting $\mathbb{E}[W_c]$ with $\mathbb{E}[W]$ in equation 2.4 we get:

$$W_l = \begin{cases} 1\mathbb{E}[W_c] & \text{with } p = 0.375 \\ 2\mathbb{E}[W_c] & \text{with } p = 0.26563 \\ 3\mathbb{E}[W_c] & \text{with } p = 0.35938 \end{cases} \quad \mathbb{E}[W_l] = -171 \tag{2.4}$$

2.4.6 House Edge

Therefore for a 0+1 a player is expected to spend 408€ while their expected loss is 330€$(-\mathbb{E}[X_l] - \mathbb{E}[W_l])$ for a House Advantage of $\approx 80.88\%$.

Compared to the analyzed traditional gambling games, this is the highest yet. However, the game also gives players some free Primogems to let them play for free. From historical data[5], Genshin Impact gives players, on average, 12593 Primogems per version, which translates to 79 free pulls or 158€.

[5] https://1drv.ms/x/s!Apq2XIWjrGXMhXFzpSHWFi57Xyjl?e=N4heJr

Adjusted for these free tokens, the adjusted cost is 172 euros, and the house edge decreases to 42.15%, meaning that for every euro spent, a player loses 0.42€.

2.5 Summary

Table 2.2: House Edges

Game	House Edge %
European Roulette	2.7
American Roulette	5.2
Liberty Bell	24.4
Mills	20.25
8-spot Keno	29.673
10-spot Keno	30.776
Genshin	80.88
Adj. Genshin	42.15

Conclusion

Our findings align with Drummond et al.'s [2] conclusion that E-I games can cause as much, if not more, harm than traditional forms of gambling.

However, it should be acknowledged that, given such odds, few players spend money with the expectation of financial returns [14]. Instead, as Jang [12] points out, "play frequency and social interaction are positively associated with the intention to make in-app purchases." In fact, it has been shown that in-game items can serve social purposes, such as displaying status or skill [7]. Additionally, RRMs were initially introduced by game studios as a cost-effective way to add variety, novelty, and replayability to games by surprising players with different items during each session. Limiting their use could stifle innovation and raise the barriers to entry for aspiring game developers.

Nevertheless, this review recognizes the urgent need for political and industry-wide reforms to mitigate the risk of individuals, particularly younger players, unknowingly engaging in gambling activities. Moreover, despite regulations in China, most Chinese companies adopt suboptimal probability disclosure policies. For example, Mihoyo uses confusing technical language and mathematical formulas that obscure the true cost of its games, making it difficult for the average player to understand.

To address these concerns, games should disclose their probabilities in simpler terms and existing regulations should be enforced more rigorously.

Appendix

A.1 Standard Banner

Table 2.3: Expected Value of 5*star item In Standard Banner

N. Pull	p	$1-p$	Cum Prob of Failure	Prob Of Success	$N \cdot \mathbb{P}(Success)$
1	0,0060	0,9940	1,0000	0,0060	0,006
2	0,0060	0,9940	0,9940	0,0060	0,012
3	0,0060	0,9940	0,9880	0,0059	0,018
4	0,0060	0,9940	0,9821	0,0059	0,024
5	0,0060	0,9940	0,9762	0,0059	0,029
6	0,0060	0,9940	0,9704	0,0058	0,035
7	0,0060	0,9940	0,9645	0,0058	0,041
8	0,0060	0,9940	0,9587	0,0058	0,046
9	0,0060	0,9940	0,9530	0,0057	0,051
10	0,0060	0,9940	0,9473	0,0057	0,057
11	0,0060	0,9940	0,9416	0,0056	0,062
12	0,0060	0,9940	0,9359	0,0056	0,067
13	0,0060	0,9940	0,9303	0,0056	0,073
14	0,0060	0,9940	0,9247	0,0055	0,078

15	0,0060	0,9940	0,9192	0,0055	0,083
16	0,0060	0,9940	0,9137	0,0055	0,088
17	0,0060	0,9940	0,9082	0,0054	0,093
18	0,0060	0,9940	0,9028	0,0054	0,097
19	0,0060	0,9940	0,8973	0,0054	0,102
20	0,0060	0,9940	0,8920	0,0054	0,107
21	0,0060	0,9940	0,8866	0,0053	0,112
22	0,0060	0,9940	0,8813	0,0053	0,116
23	0,0060	0,9940	0,8760	0,0053	0,121
24	0,0060	0,9940	0,8707	0,0052	0,125
25	0,0060	0,9940	0,8655	0,0052	0,130
26	0,0060	0,9940	0,8603	0,0052	0,134
27	0,0060	0,9940	0,8552	0,0051	0,139
28	0,0060	0,9940	0,8500	0,0051	0,143
29	0,0060	0,9940	0,8449	0,0051	0,147
30	0,0060	0,9940	0,8399	0,0050	0,151
31	0,0060	0,9940	0,8348	0,0050	0,155
32	0,0060	0,9940	0,8298	0,0050	0,159
33	0,0060	0,9940	0,8248	0,0049	0,163
34	0,0060	0,9940	0,8199	0,0049	0,167
35	0,0060	0,9940	0,8150	0,0049	0,171
36	0,0060	0,9940	0,8101	0,0049	0,175
37	0,0060	0,9940	0,8052	0,0048	0,179
38	0,0060	0,9940	0,8004	0,0048	0,182
39	0,0060	0,9940	0,7956	0,0048	0,186
40	0,0060	0,9940	0,7908	0,0047	0,190

41	0,0060	0,9940	0,7861	0,0047	0,193
42	0,0060	0,9940	0,7813	0,0047	0,197
43	0,0060	0,9940	0,7767	0,0047	0,200
44	0,0060	0,9940	0,7720	0,0046	0,204
45	0,0060	0,9940	0,7674	0,0046	0,207
46	0,0060	0,9940	0,7628	0,0046	0,211
47	0,0060	0,9940	0,7582	0,0045	0,214
48	0,0060	0,9940	0,7536	0,0045	0,217
49	0,0060	0,9940	0,7491	0,0045	0,220
50	0,0060	0,9940	0,7446	0,0045	0,223
51	0,0060	0,9940	0,7401	0,0044	0,226
52	0,0060	0,9940	0,7357	0,0044	0,230
53	0,0060	0,9940	0,7313	0,0044	0,233
54	0,0060	0,9940	0,7269	0,0044	0,236
55	0,0060	0,9940	0,7225	0,0043	0,238
56	0,0060	0,9940	0,7182	0,0043	0,241
57	0,0060	0,9940	0,7139	0,0043	0,244
58	0,0060	0,9940	0,7096	0,0043	0,247
59	0,0060	0,9940	0,7054	0,0042	0,250
60	0,0060	0,9940	0,7011	0,0042	0,252
61	0,0060	0,9940	0,6969	0,0042	0,255
62	0,0060	0,9940	0,6927	0,0042	0,258
63	0,0060	0,9940	0,6886	0,0041	0,260
64	0,0060	0,9940	0,6845	0,0041	0,263
65	0,0060	0,9940	0,6803	0,0041	0,265
66	0,0060	0,9940	0,6763	0,0041	0,268

67	0,0060	0,9940	0,6722	0,0040	0,270
68	0,0060	0,9940	0,6682	0,0040	0,273
69	0,0060	0,9940	0,6642	0,0040	0,275
70	0,0060	0,9940	0,6602	0,0040	0,277
71	0,0060	0,9940	0,6562	0,0039	0,280
72	0,0060	0,9940	0,6523	0,0039	0,282
73	0,0060	0,9940	0,6484	0,0039	0,284
74	0,0645	0,9355	0,6445	0,0415	3,075
75	0,1229	0,8771	0,6029	0,0741	5,559
76	0,1814	0,8186	0,5288	0,0959	7,291
77	0,2399	0,7601	0,4329	0,1038	7,996
78	0,2984	0,7016	0,3290	0,0982	7,657
79	0,3568	0,6432	0,2309	0,0824	6,508
80	0,4153	0,5847	0,1485	0,0617	4,933
81	0,4738	0,5262	0,0868	0,0411	3,332
82	0,5322	0,4678	0,0457	0,0243	1,994
83	0,5907	0,4093	0,0214	0,0126	1,048
84	0,6492	0,3508	0,0087	0,0057	0,477
85	0,7076	0,2924	0,0031	0,0022	0,185
86	0,7661	0,2339	0,0009	0,0007	0,059
87	0,8246	0,1754	0,0002	0,0002	0,015
88	0,8831	0,1169	0,0000	0,0000	0,003
89	0,9415	0,0585	0,0000	0,0000	0,000
90	1,0000	0,0000	0,0000	0,0000	0,000
					E[X]
					62,3

A.2 Limited Weapon Banner

Table 2.4: Expected Value of 5*star item In Limited Weapon Banner

N. Pull	p	$1-p$	Prob of Failure	Prob Of Success	$N \cdot \mathbb{P}(Success)$
1	0,006	0,994	1	0,006	0,006
2	0,006	0,994	0,994	0,005964	0,011928
3	0,006	0,994	0,988036	0,005928216	0,017784648
4	0,006	0,994	0,982107784	0,005892647	0,023570587
5	0,006	0,994	0,976215137	0,005857291	0,029286454
6	0,006	0,994	0,970357846	0,005822147	0,034932882
7	0,006	0,994	0,964535699	0,005787214	0,040510499
8	0,006	0,994	0,958748485	0,005752491	0,046019927
9	0,006	0,994	0,952995994	0,005717976	0,051461784
10	0,006	0,994	0,947278018	0,005683668	0,056836681
11	0,006	0,994	0,94159435	0,005649566	0,062145227
12	0,006	0,994	0,935944784	0,005615669	0,067388024
13	0,006	0,994	0,930329115	0,005581975	0,072565671
14	0,006	0,994	0,924747141	0,005548483	0,07767876
15	0,006	0,994	0,919198658	0,005515192	0,082727879
16	0,006	0,994	0,913683466	0,005482101	0,087713613
17	0,006	0,994	0,908201365	0,005449208	0,092636539
18	0,006	0,994	0,902752157	0,005416513	0,097497233
19	0,006	0,994	0,897335644	0,005384014	0,102296263
20	0,006	0,994	0,89195163	0,00535171	0,107034196
21	0,006	0,994	0,88659992	0,0053196	0,11171159

22	0,006	0,994	0,881280321	0,005287682	0,116329002
23	0,006	0,994	0,875992639	0,005255956	0,120886984
24	0,006	0,994	0,870736683	0,00522442	0,125386082
25	0,006	0,994	0,865512263	0,005193074	0,129826839
26	0,006	0,994	0,860319189	0,005161915	0,134209794
27	0,006	0,994	0,855157274	0,005130944	0,138535478
28	0,006	0,994	0,850026331	0,005100158	0,142804424
29	0,006	0,994	0,844926173	0,005069557	0,147017154
30	0,006	0,994	0,839856616	0,00503914	0,151174191
31	0,006	0,994	0,834817476	0,005008905	0,155276051
32	0,006	0,994	0,829808571	0,004978851	0,159323246
33	0,006	0,994	0,82482972	0,004948978	0,163316284
34	0,006	0,994	0,819880741	0,004919284	0,167255671
35	0,006	0,994	0,814961457	0,004889769	0,171141906
36	0,006	0,994	0,810071688	0,00486043	0,174975485
37	0,006	0,994	0,805211258	0,004831268	0,178756899
38	0,006	0,994	0,80037999	0,00480228	0,182486638
39	0,006	0,994	0,79557771	0,004773466	0,186165184
40	0,006	0,994	0,790804244	0,004744825	0,189793019
41	0,006	0,994	0,786059419	0,004716357	0,193370617
42	0,006	0,994	0,781343062	0,004688058	0,196898452
43	0,006	0,994	0,776655004	0,00465993	0,200376991
44	0,006	0,994	0,771995074	0,00463197	0,203806699
45	0,006	0,994	0,767363103	0,004604179	0,207188038
46	0,006	0,994	0,762758925	0,004576554	0,210521463
47	0,006	0,994	0,758182371	0,004549094	0,213807429

48	0,006	0,994	0,753633277	0,0045218	0,217046384
49	0,006	0,994	0,749111477	0,004494669	0,220238774
50	0,006	0,994	0,744616808	0,004467701	0,223385043
51	0,006	0,994	0,740149108	0,004440895	0,226485627
52	0,006	0,994	0,735708213	0,004414249	0,229540962
53	0,006	0,994	0,731293964	0,004387764	0,23255148
54	0,006	0,994	0,7269062	0,004361437	0,235517609
55	0,006	0,994	0,722544763	0,004335269	0,238439772
56	0,006	0,994	0,718209494	0,004309257	0,24131839
57	0,006	0,994	0,713900237	0,004283401	0,244153881
58	0,006	0,994	0,709616836	0,004257701	0,246946659
59	0,006	0,994	0,705359135	0,004232155	0,249697134
60	0,006	0,994	0,70112698	0,004206762	0,252405713
61	0,006	0,994	0,696920218	0,004181521	0,2550728
62	0,006	0,994	0,692738697	0,004156432	0,257698795
63	0,076	0,924	0,688582265	0,052332252	3,296931883
64	0,146	0,854	0,636250012	0,092892502	5,945120116
65	0,216	0,784	0,543357511	0,117365222	7,628739449
66	0,286	0,714	0,425992288	0,121833794	8,041030435
67	0,356	0,644	0,304158494	0,108280424	7,254788396
68	0,426	0,574	0,19587807	0,083444058	5,674195933
69	0,496	0,504	0,112434012	0,05576727	3,847941634
70	0,566	0,434	0,056666742	0,032073376	2,245136324
71	0,636	0,364	0,024593366	0,015641381	1,110538039
72	0,706	0,294	0,008951985	0,006320102	0,455047315
73	0,776	0,224	0,002631884	0,002042342	0,149090946

74	0,846	0,154	0,000589542	0,000498752	0,036907684
75	0,916	0,084	9,07895E-05	8,31631E-05	0,006237236
76	0,986	0,014	7,62631E-06	7,51955E-06	0,000571486
77	1	0	1,06768E-07	1,06768E-07	8,22117E-06
78	1	0	0	0	0
79	1	0	0	0	0
80	1	0	0	0	0
					E[X]
					54,90

A.3 Keno Payout

Table 3.2 Statistics for 8- and 10-spot keno games on the Las Vegas Strip, January 2008. MAP is maximum aggregate payout, HA is house advantage, and SD is standard deviation. (Only eight of the 23 casinos surveyed offered live keno.)

8 spots

casino	min bet (=: 1 unit)	MAP (in units)	units paid out per unit bet* for catching 0–3	4	5	6	7	8	HA(%)	SD
Excalibur	$1	100,000	0	0	9	90	1,500	20,000	29.468	46.037
Harrah's et al.**	$2	100,000	0	0	7	80	1,500	30,000	31.149	65.480
Imperial Palace	$1	200,000	0	1	5	80	1,480	25,000	29.153	55.522
Sahara	$1	100,000	0	0	9	90	1,500	25,000	27.295	55.652
Treasure Island	$1	100,000	0	0	8	90	1,500	20,000	31.298	46.034
									29.673	

10 spots

casino	min bet (=: 1 unit)	MAP (in units)	units paid out per unit bet* for catching 0–4	5	6	7	8	9	10	HA(%)	SD
Excalibur	$2	50,000	0	2	20	130	1,000	4,000	25,000	29.540	18.303
Harrah's et al.**	$2	100,000	0	0	23	150	1,000	5,000	40,000	32.379	22.580
Imperial Palace	$1	200,000	0	1	25	125	1,000	4,000	30,000	29.693	19.138
Sahara	$2	50,000	0	2	20	130	1,000	4,000	25,000	29.540	18.303
Treasure Island	$1	100,000	0	1	22	132	960	3,800	25,000	32.729	17.792
										30.776	

*regardless of bet size, number of units paid out cannot exceed maximum aggregate payout
**includes Bally's, Caesars Palace, Flamingo, and Harrah's

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